Mitigating Non-linear DAC Glitches Using Dither in Closed-loop Nano-positioning Applications

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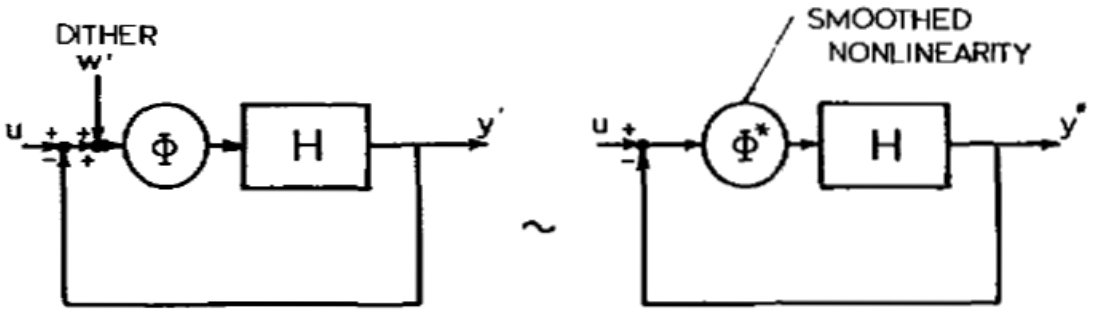
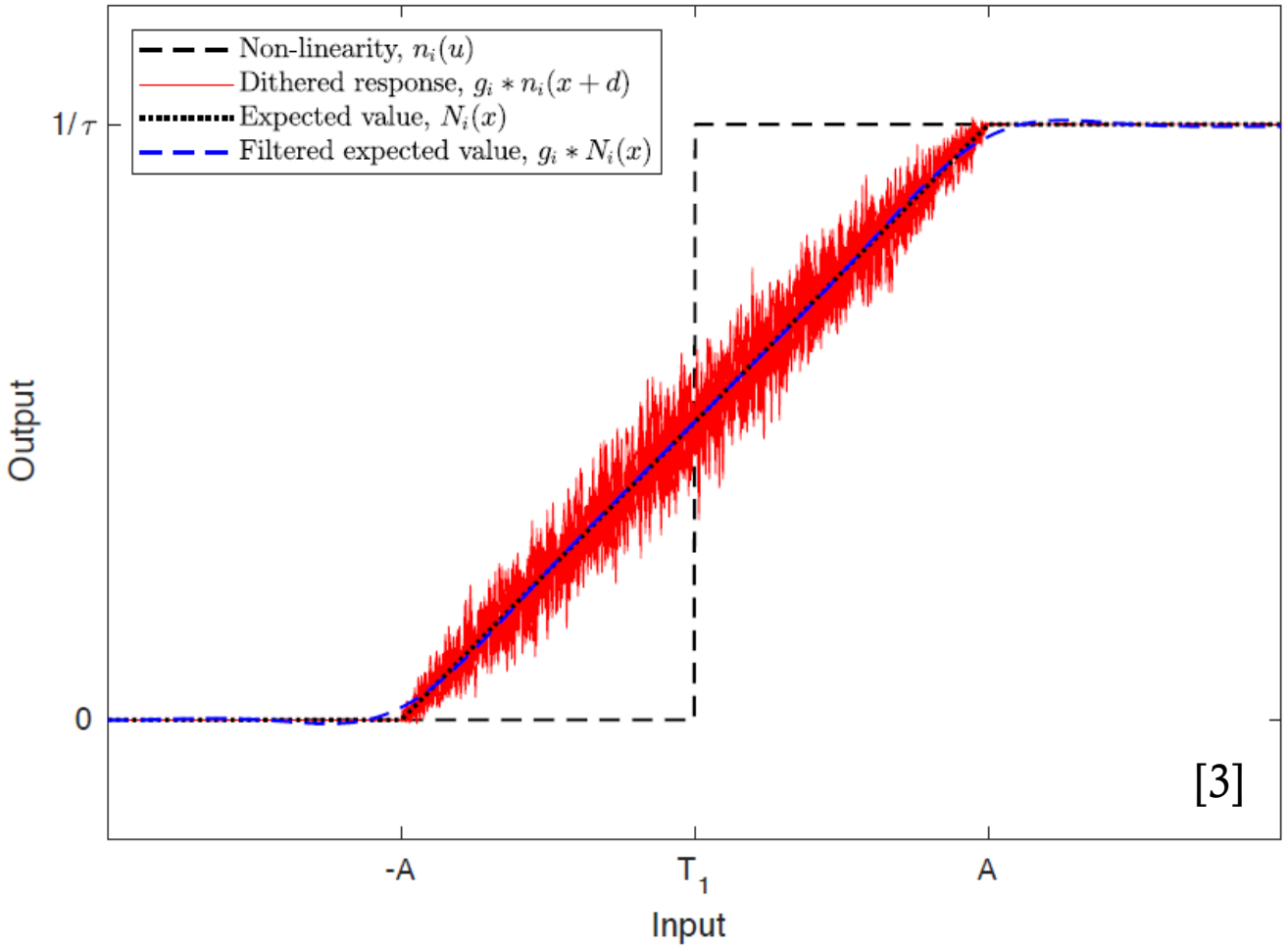
WHAT IS DITHERING?

- Dithering: is a process by which a form of noise is intentionally applied to a signal in order to randomize the quantization error (e.g. due to intended resolution reduction) [1].



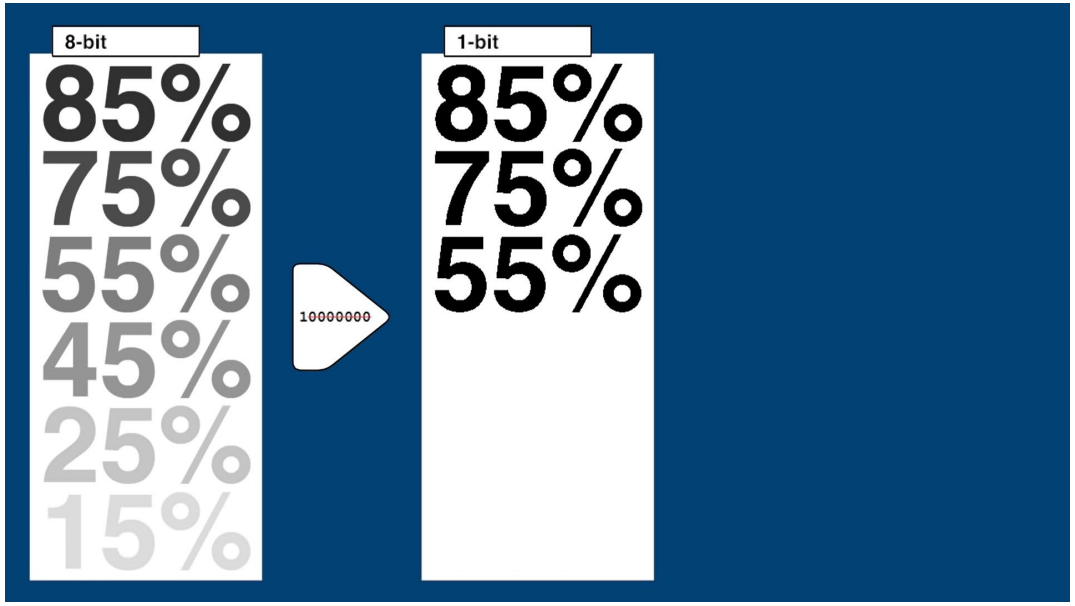
[2]



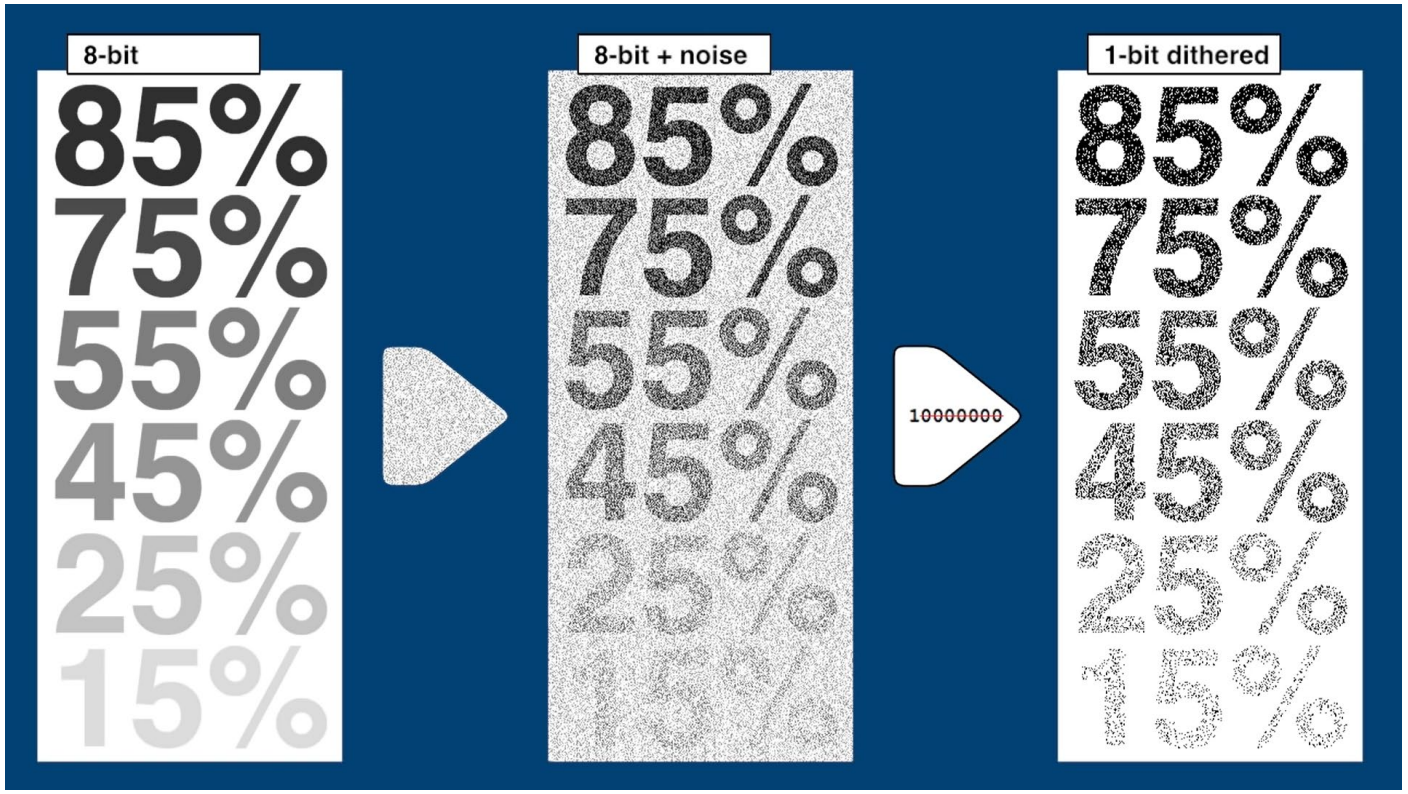
[3]

DITHERING EXAMPLE

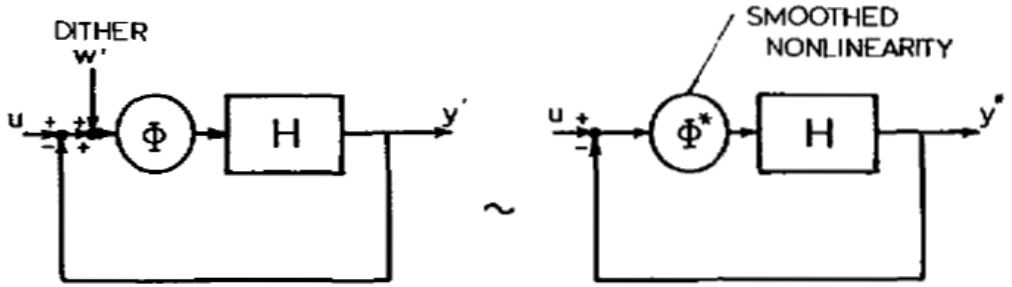
Ideal Quantization



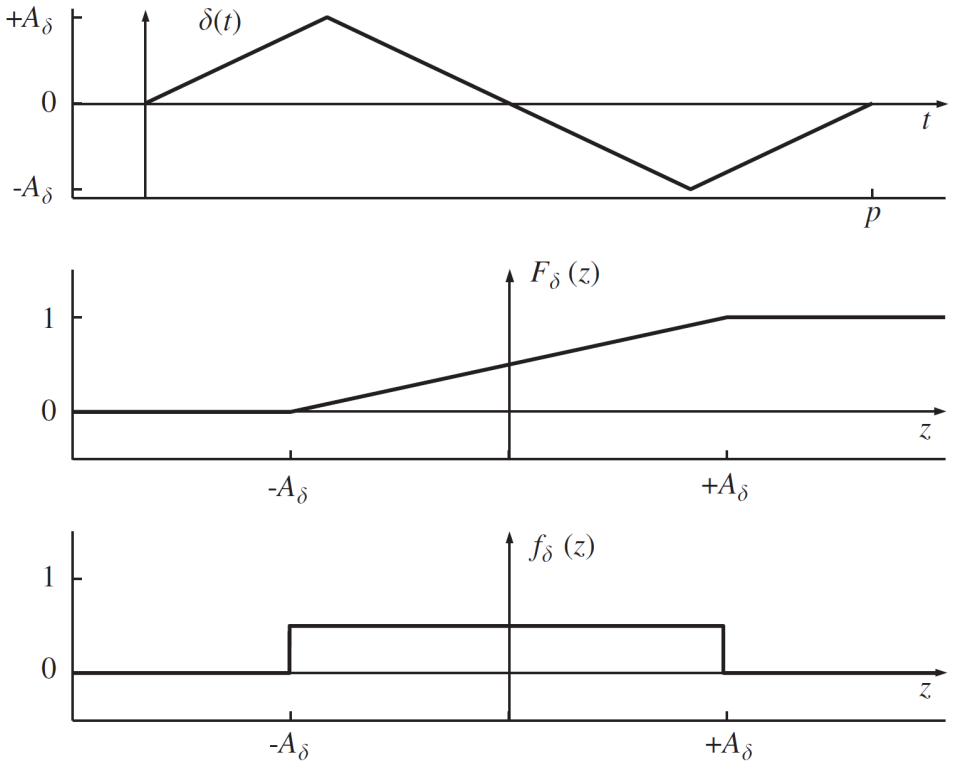
Dithered system



DITHER CHARACTERISTICS



[2]

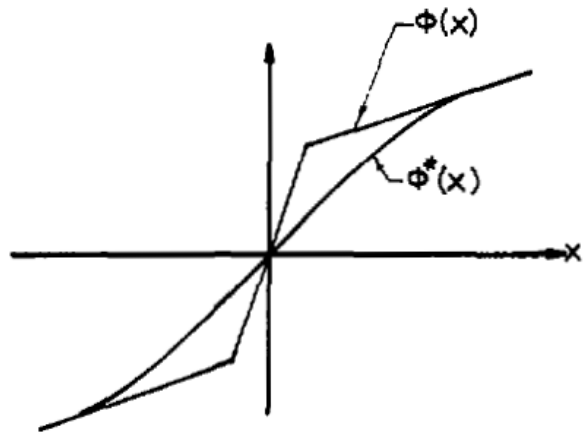


[4]

$$N(z) \triangleq \int_{\mathbb{R}} n(z+\xi)\,\mathrm{d}F_\delta(\xi)$$

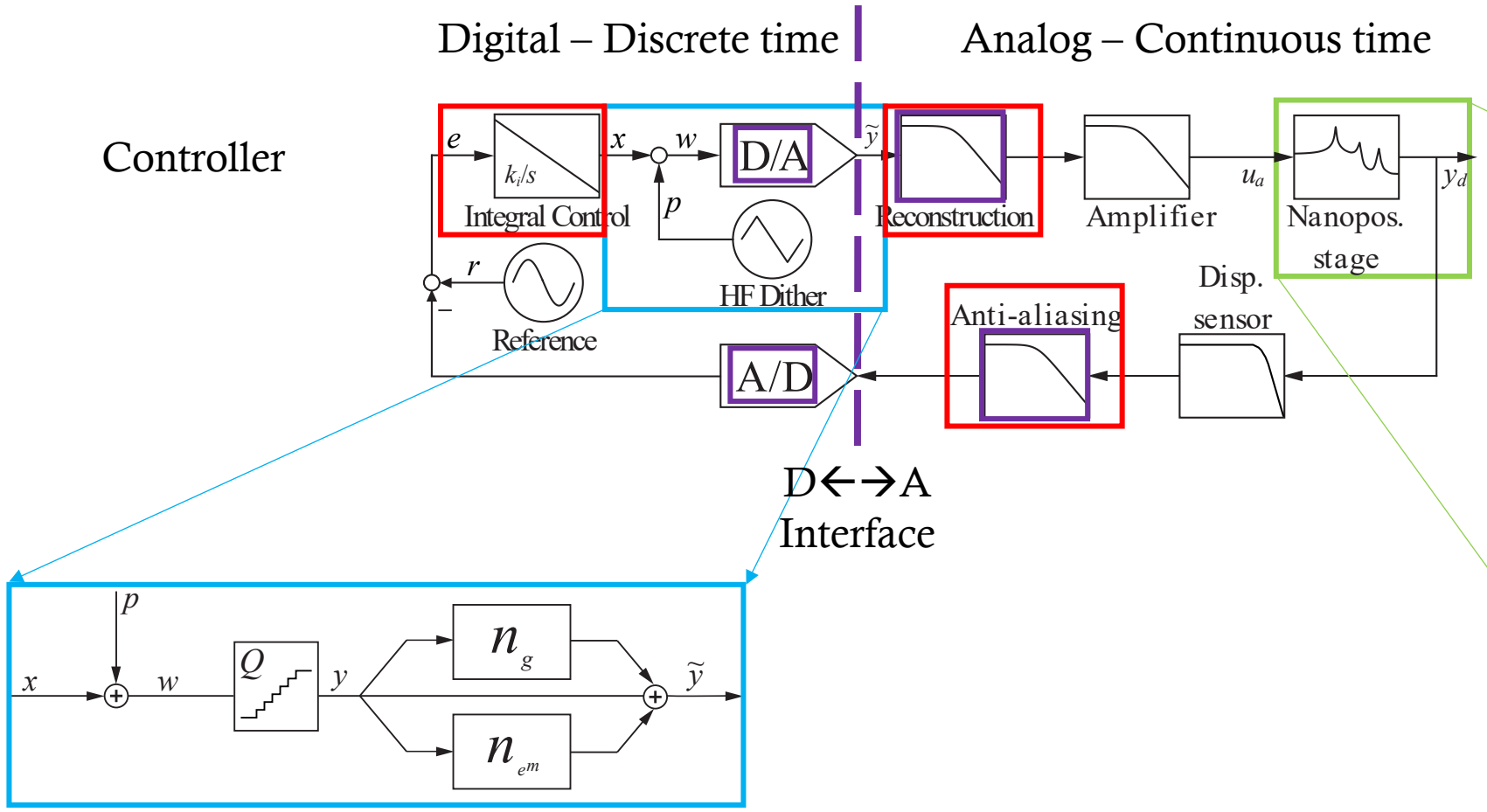
$$= \frac{1}{p}\int_{[0,p)} n(z+\delta(s))\,\mathrm{d}s$$

$$= \int_{\mathbb{R}} n(z+\xi)\,f_\delta(\xi)\,\mathrm{d}\xi$$

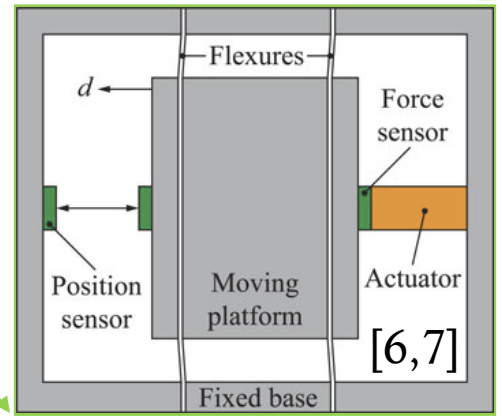


CLOSED-LOOP NANO-POSITIONING SYSTEM

[5]



DAC model accounting for glitched and INL non-linearities

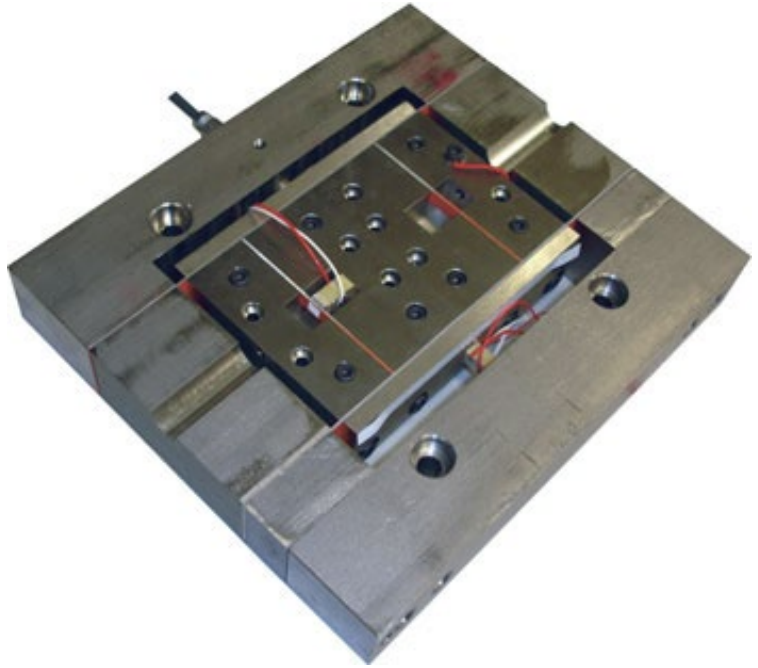


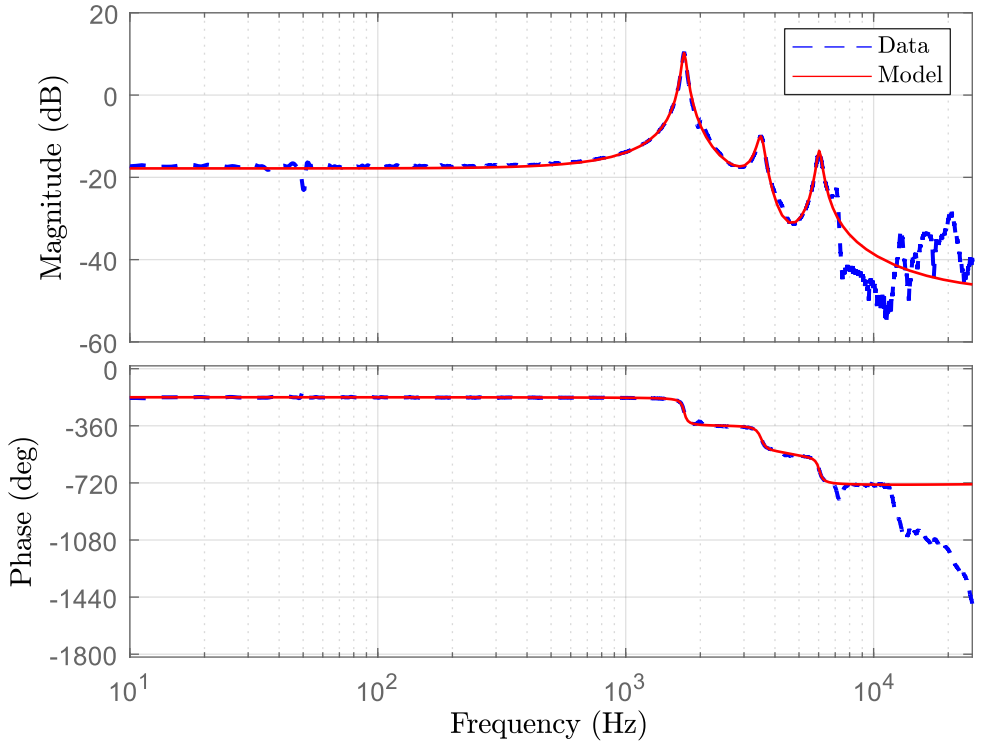
Nano-positioning mechatronic system

NANO-POSITIONING STAGE MODEL

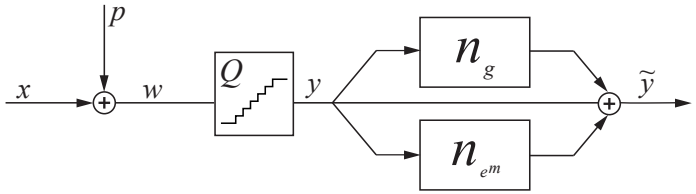
- Response fitted to an Ultra high precision

nano-positioning stage mechatronic system [6,7]





NON-LINEAR DAC MODEL



DAC model accounting for glitched and INL non-linearities for a 16-bit Texas Instruments DAC8544.

Ideal Quantization

$$Q(w) = \delta T(w) = \delta \left\lfloor \frac{w}{\delta} + \frac{1}{2} \right\rfloor$$

$$\delta = \frac{\Delta}{2^B - 1}$$

$$n_{em} = \delta \mathrm{INL}(w)$$

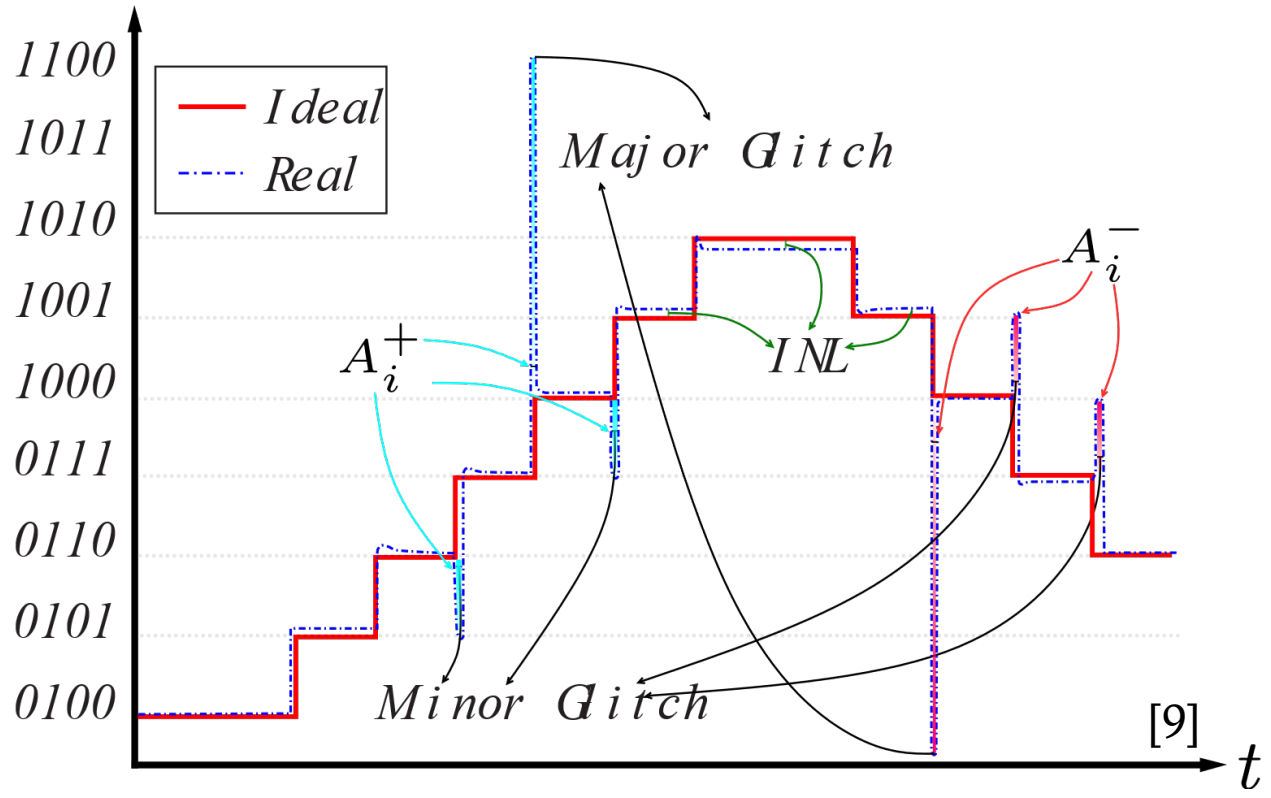
$$\mathrm{INL}(w) \triangleq \frac{\tilde{y}(w) - \delta T(w)}{\delta}$$

Element mismatch Nonlinearity [8]

$$n_g(w(t)) \triangleq \sum_{i=1}^{N_T} n_{g_i}(w(t)) \triangleq \sum_{i=1}^{N_T} A_i^{\pm}(w(t))\delta(w(t) - T_i).$$

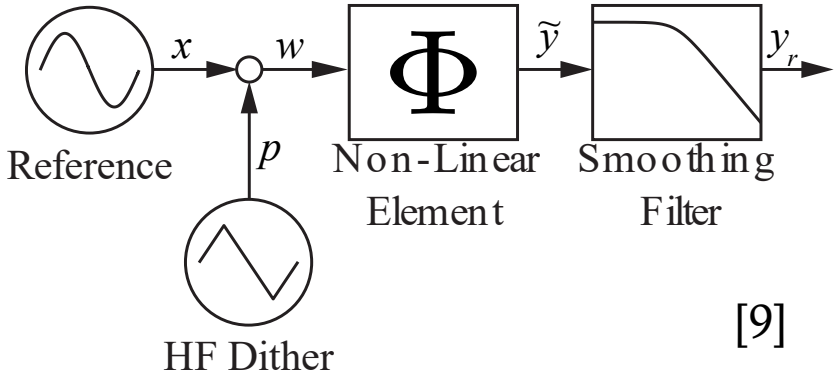
$$A_i^{\pm}(w(t)) \triangleq \begin{cases} 0 & w(t-\tau) = w(t) \\ A_i^- & w(t-\tau) > w(t) \\ A_i^+ & w(t-\tau) < w(t) \end{cases}$$

Glitch [3] Nonlinearity

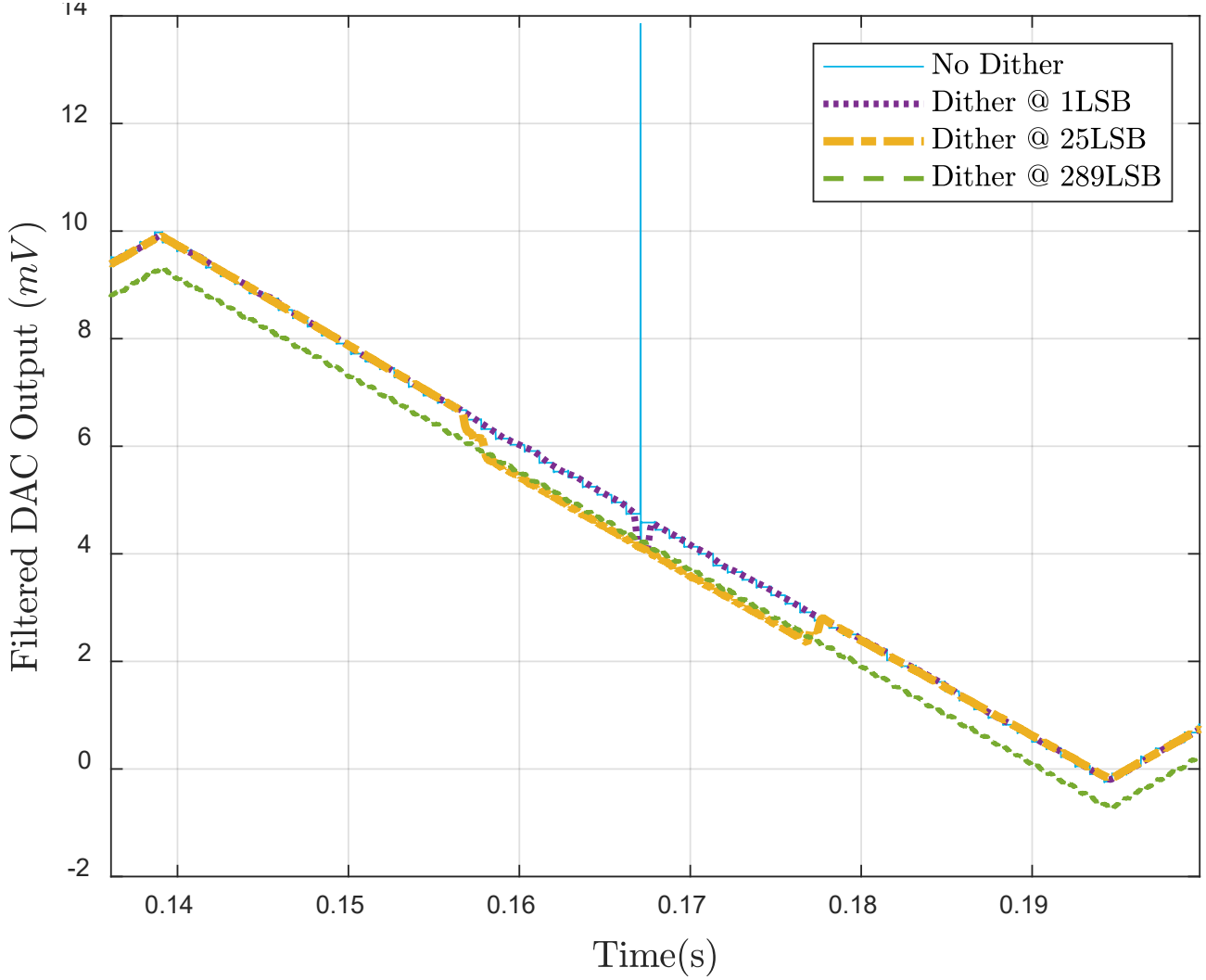


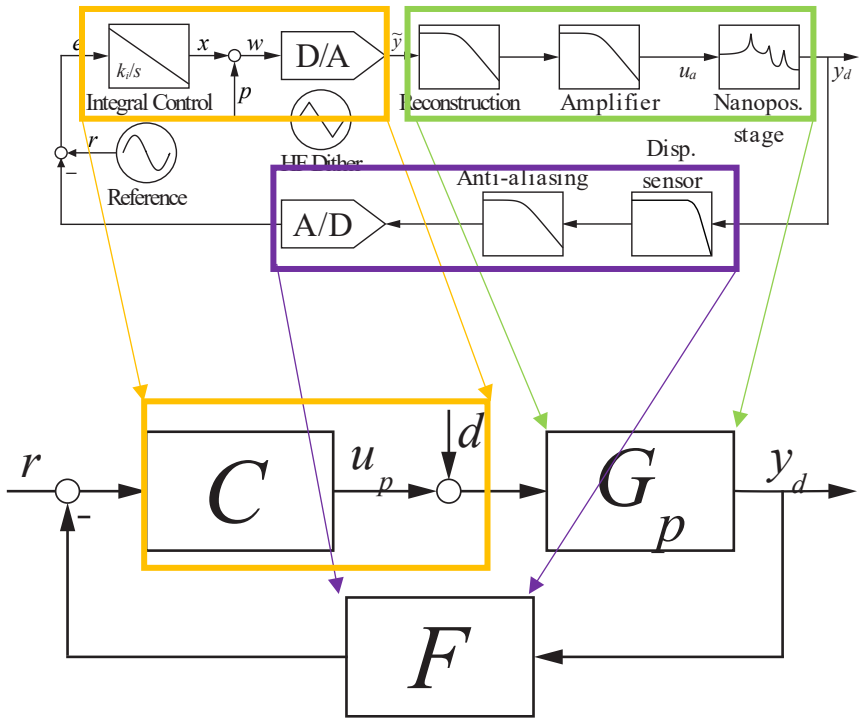
A GLITCH IN OPEN-LOOP

- Smoothed glitch due to periodic dither injection and filtering.



[9]





Nominal Design (Nyquist Criterion)

$$C_i(s) = \frac{\omega_c^2}{s^2 + \sqrt{2}\omega_c s + \omega_c^2}\frac{k_i}{s}$$

Damping Integral Incorporated Design

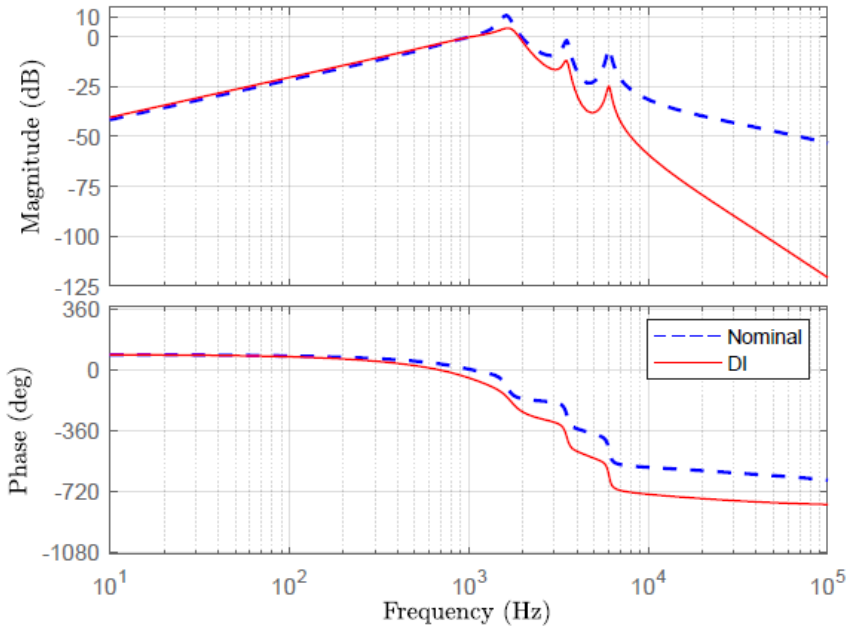
$$C_i(s) = \frac{k_i}{s}$$

$$y_d = r \Rightarrow T(s) = 1$$

$$J(\theta_c) = \| 1 - |T(j\omega;\theta_c)| \|_2$$

$$\theta_c = [\omega_c \; k_i]^T$$

$$\theta_c^* = \operatorname{argmin}_{\theta_c} J(\theta_c) : \mathrm{Re}\{\lambda_i\} \in \mathbb{R}_-$$



Performance Measure *D(s)* :

D(s) the transfer function from a disturbance ***d*** to the plant displacement ***y_d***:

$$y_d = S(s)G_p(s)d = D(s)d$$

S(s) the sensitivity of plant input ***u_p*** to a disturbance ***d*** :

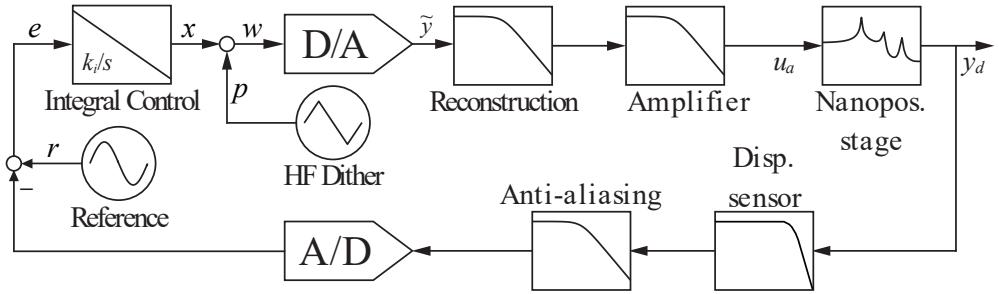
$$u_p = (1 + L(s))^{-1} d = S(s)d$$

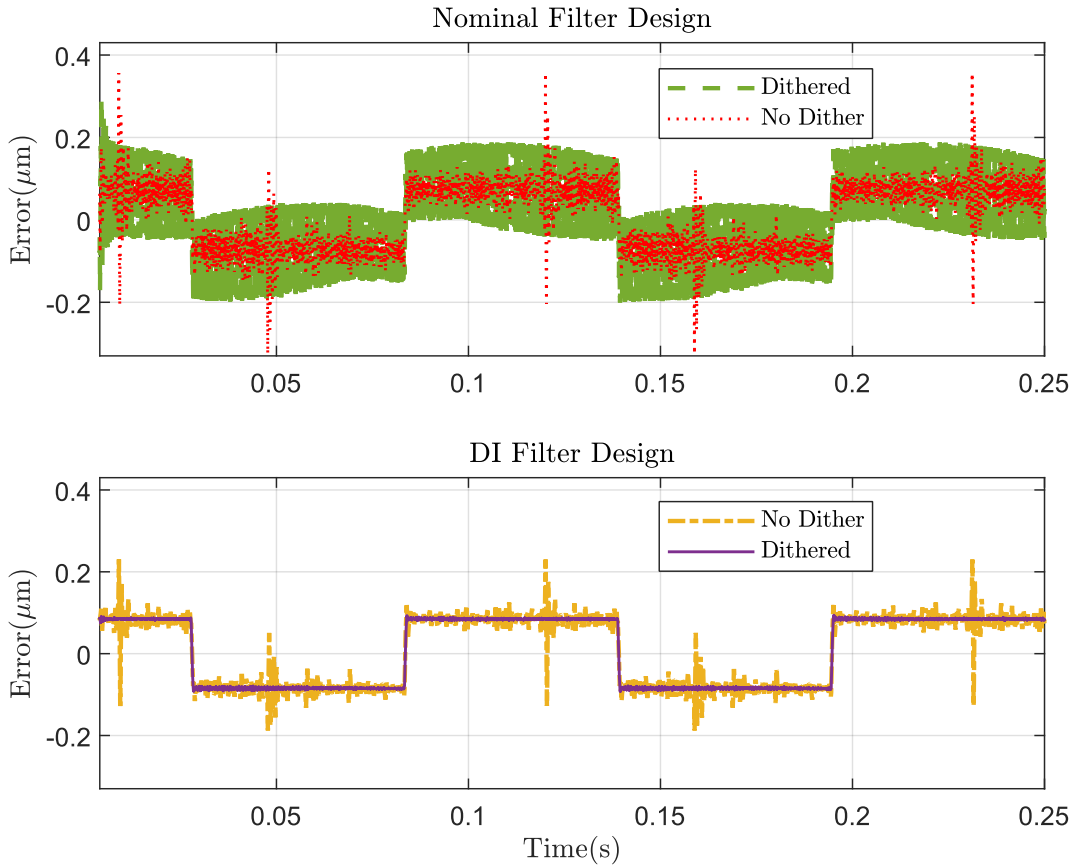
L(s) loop transfer function, breaking the loop at ***u_p***:

$$L(s) = C(s)F(s)G_p(s)$$

GLITCH DISTURBANCE IN CLOSED-LOOP

- Tracking error ***e*** due to a glitch disturbance smoothed by periodic dither injection and filtering.

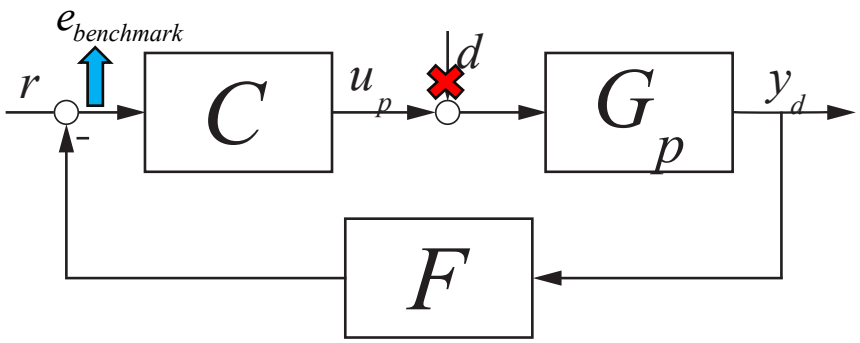


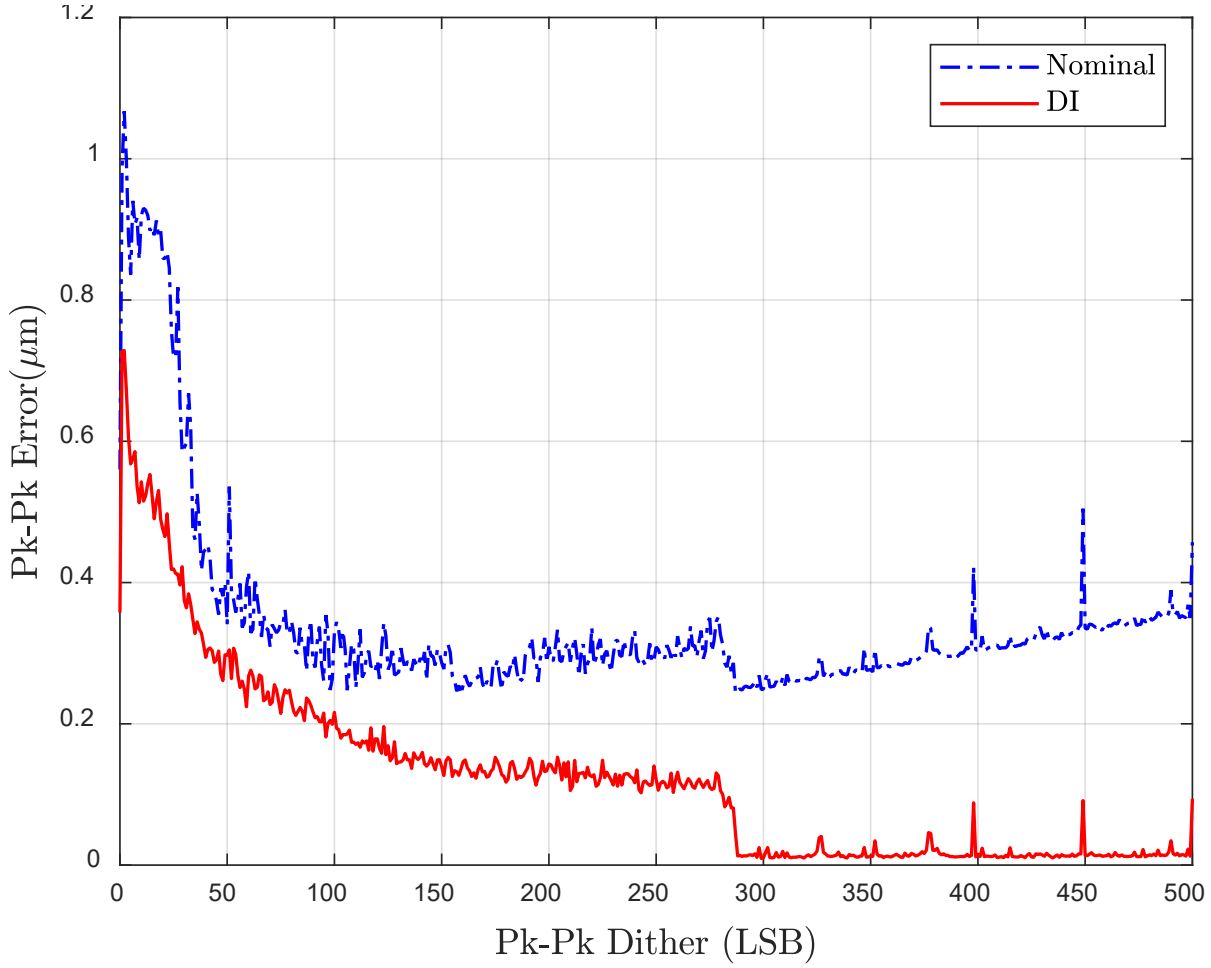


GLITCH DISTURBANCE IN CLOSED-LOOP

- Residual tracking error e_r due to a glitch disturbance smoothed by periodic dither injection and filtering.

$$e_r = e - e_{benchmark}$$





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THANK YOU