A Criterion for a Sufficiently Large Dither Amplitude to Mitigate Non-linear Glitches

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Research fellow: Ahmad Faza (UiS)

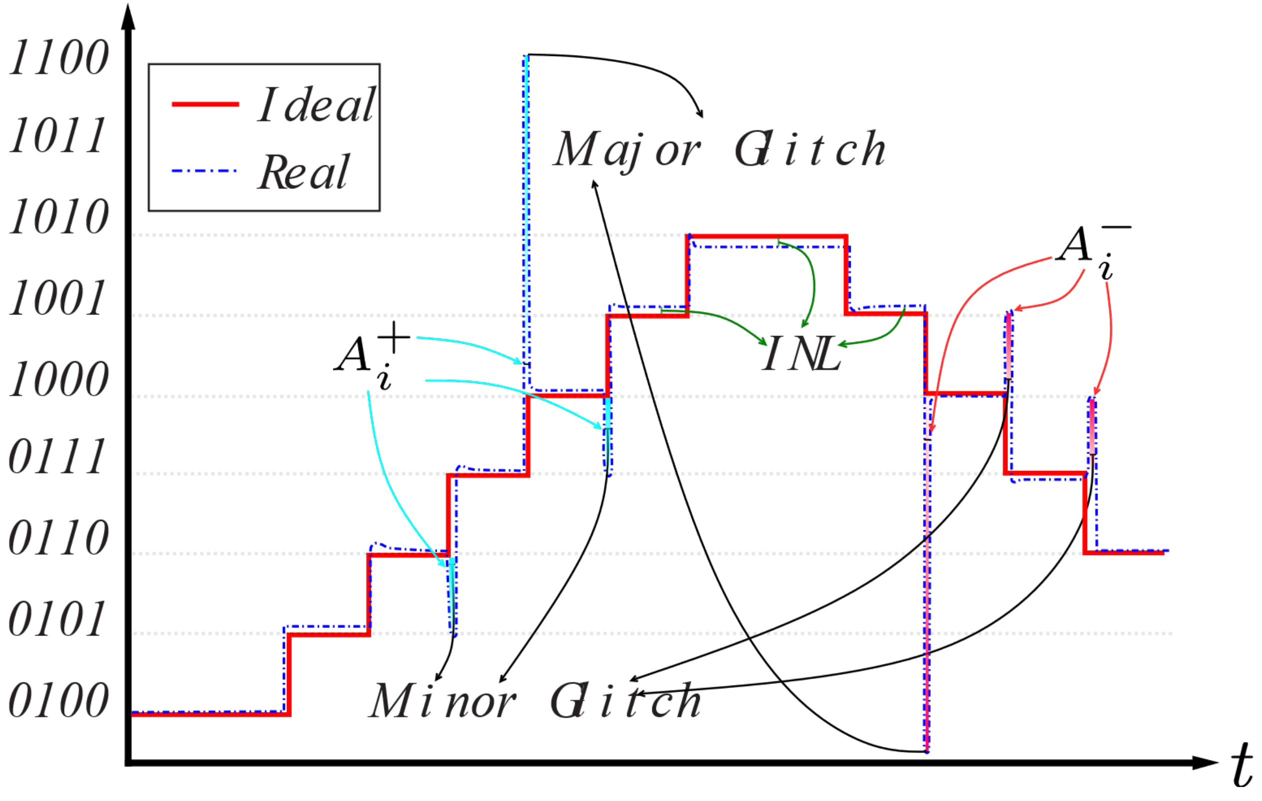
Main supervisor: Arnfinn A. Eielsen (UiS)

Co- supervisor: John-Josef Leth (AAU)

GLITCHED DISTURBANCES

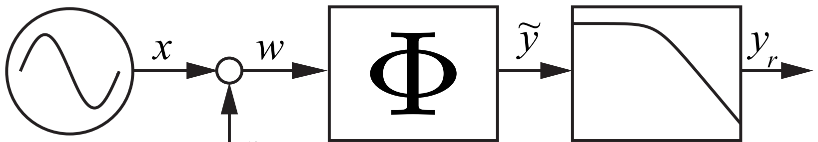
- Glitches are unwanted, transient disturbances of short duration, often resembling spikes in a waveform. They are known to occur in a myriad of **switched-mode** systems such as:
- Multi-level power converters
- Systems with friction
- Digital-to-analogue converters (DACs)

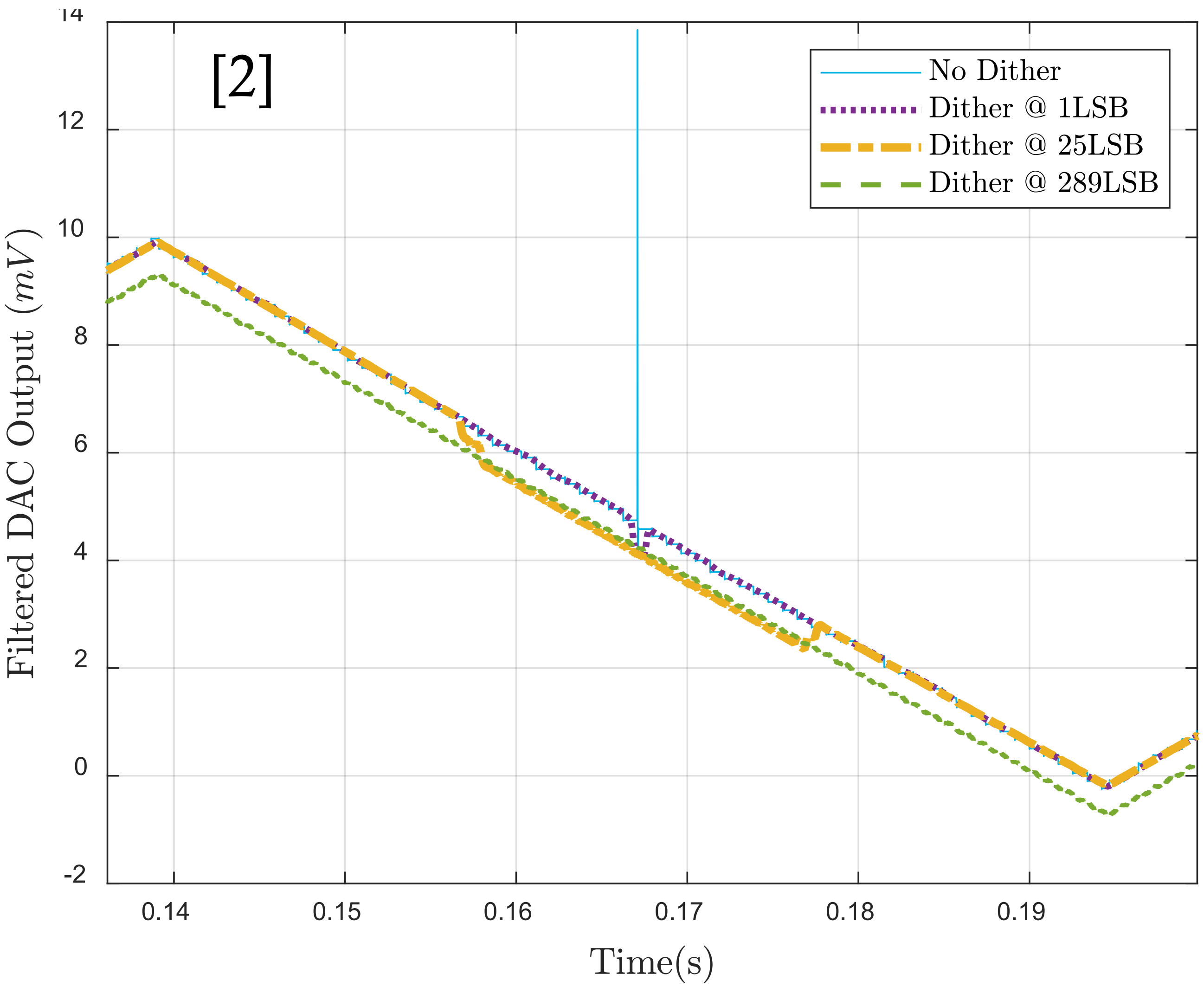
In the context of DACs:



DITHERING AND FILTERING A GLITCH IN OPEN-LOOP [1]

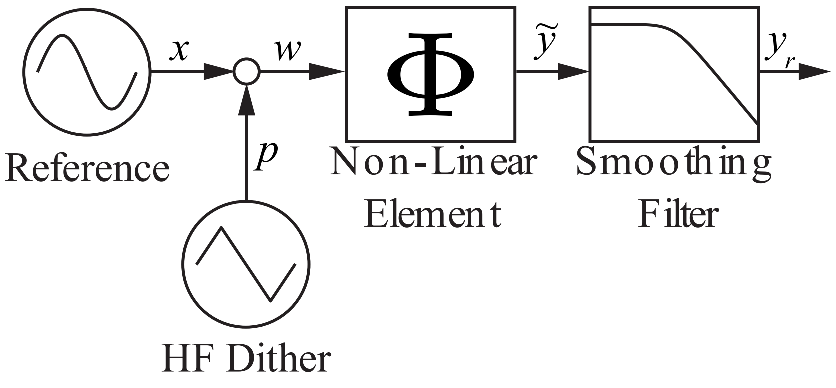
- Smoothed glitch due to periodic dither injection and filtering.

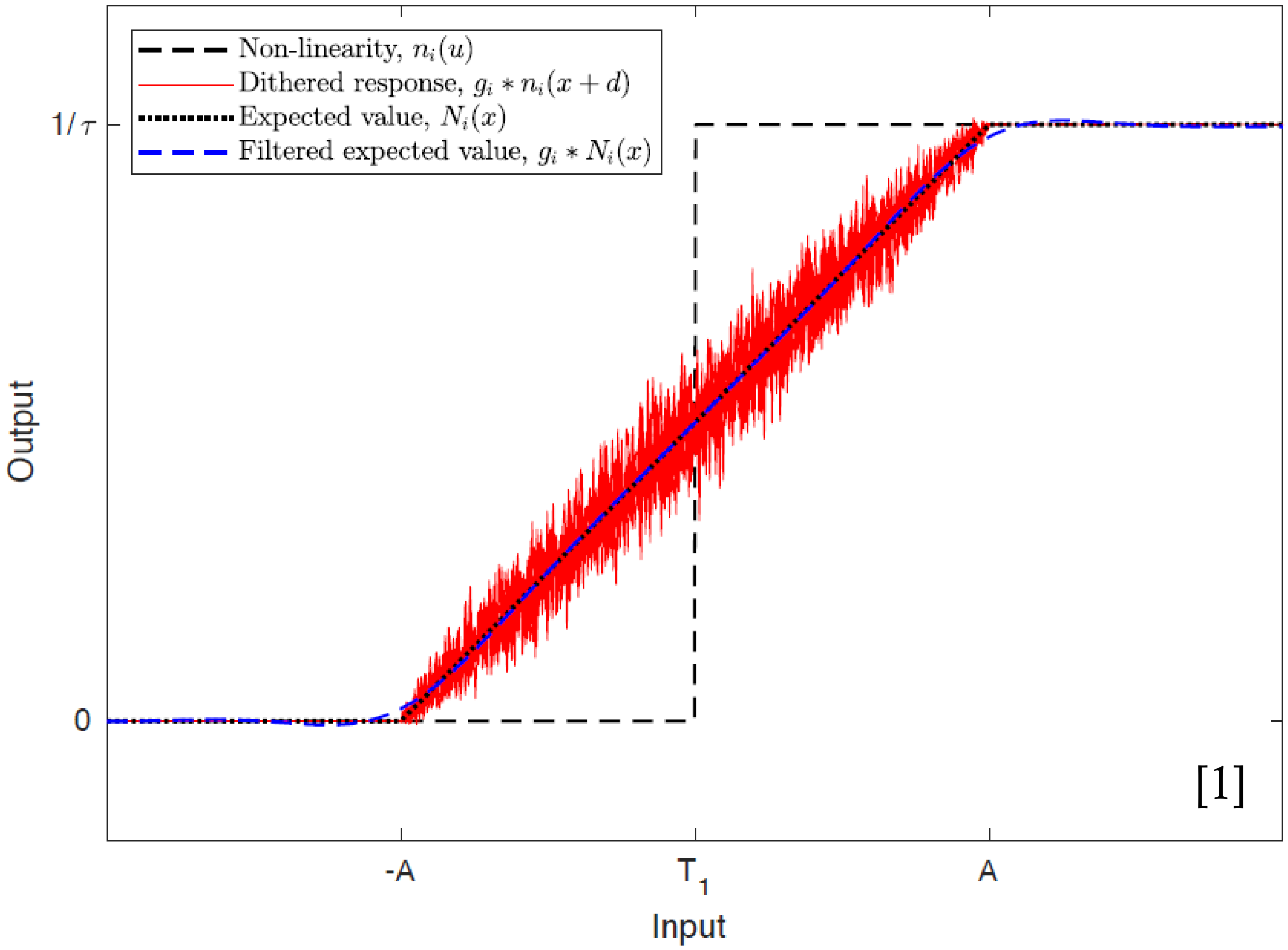




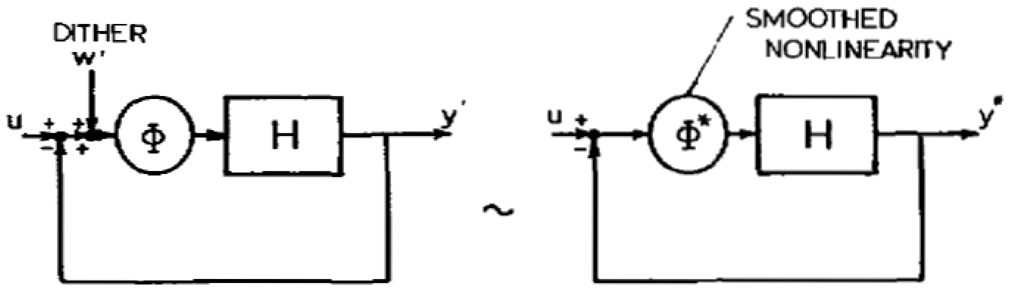
WHAT IS DITHERING?

- Dithering: is a process by which a form of noise is intentionally applied to a signal in order to randomize the quantization error (e.g. due to intended resolution reduction) [3].





DITHER CHARACTERISTICS



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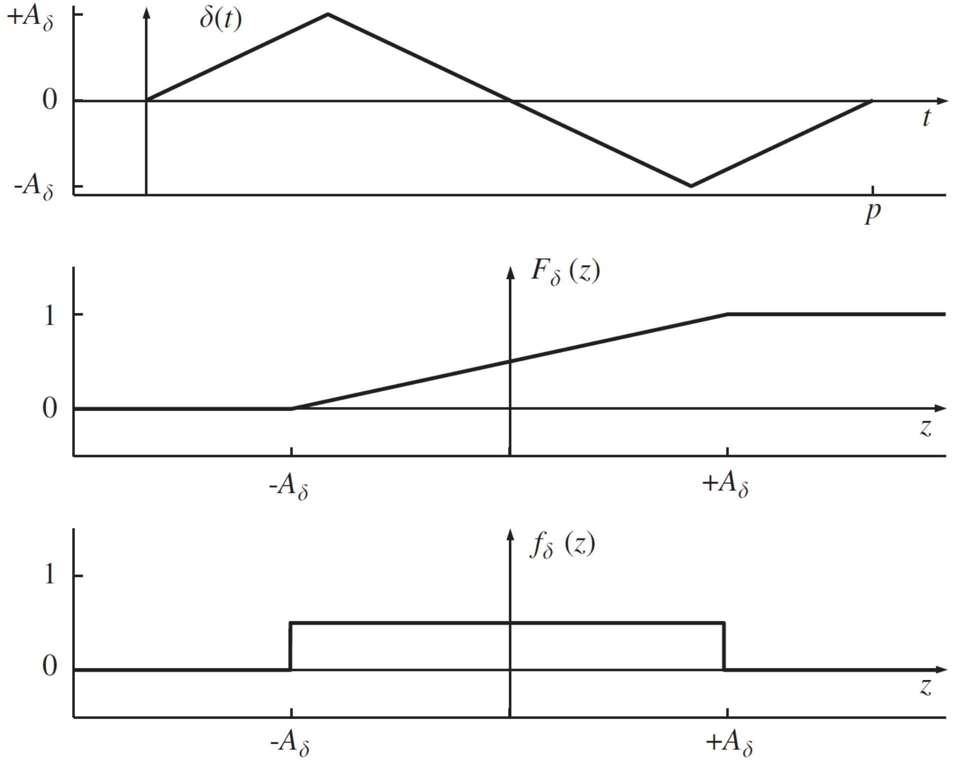
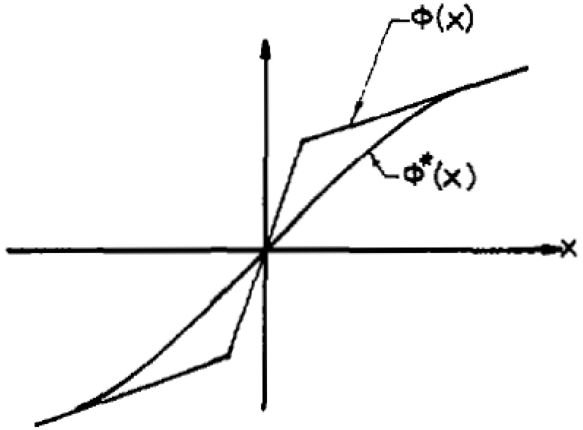


[5]

$$N(z) \triangleq \int_{\mathbb{R}} n(z+\xi)\,\mathrm{d}F_\delta(\xi)$$

$$= \int_{\mathbb{R}} n(z+\xi)\,f_\delta(\xi)\,\mathrm{d}\xi$$

$$= \frac{1}{p}\int_{[0,p)} n(z+\delta(s))\,\mathrm{d}s$$



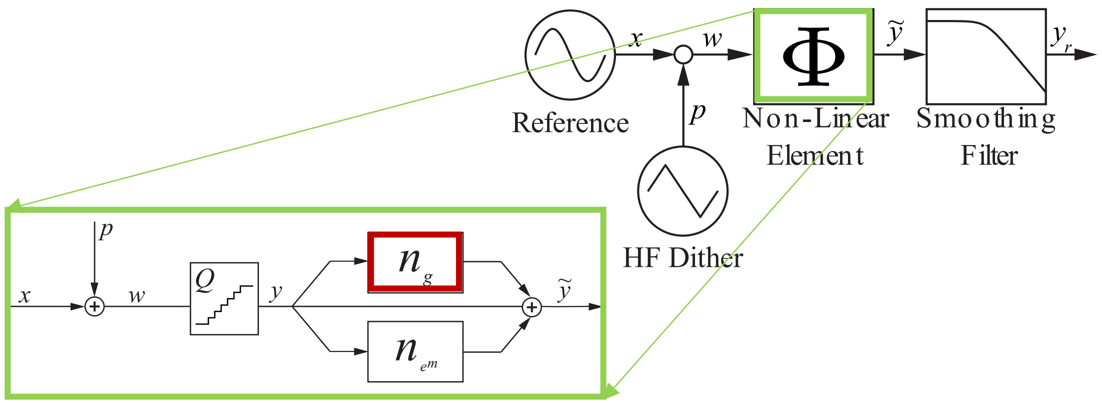
A PROPOSED GLITCH MODEL

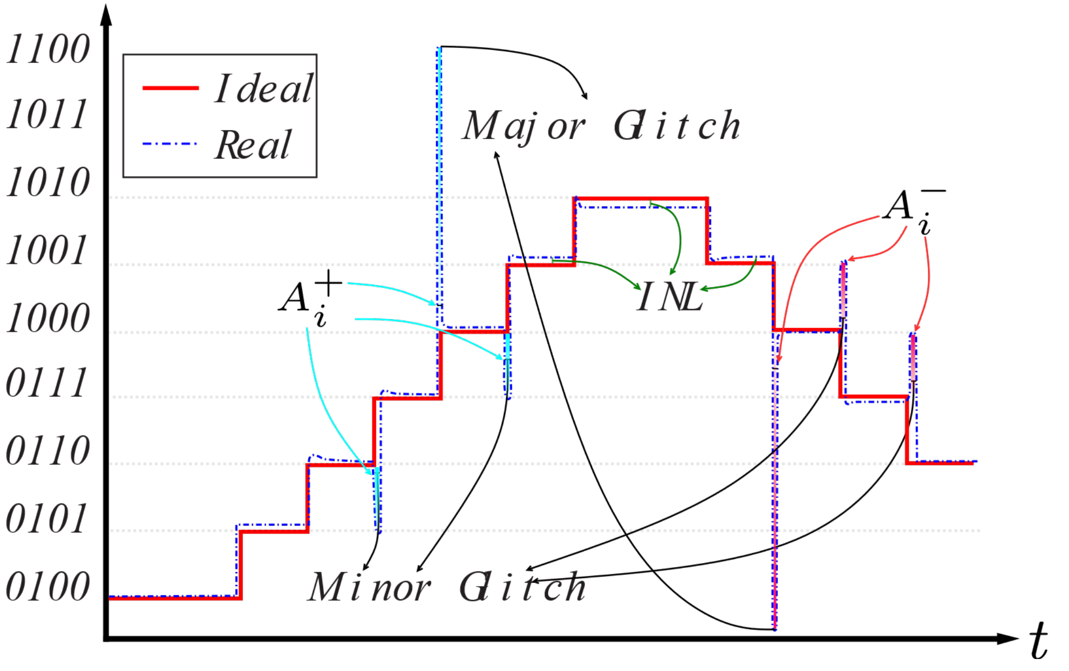
Let $\tilde{y}(t) = \phi(w(t))$. we may write $\phi(w(t)) = n(w(t)) + n_g(w(t))$ with $n_g(w(t))$ describing the effect of glitches and $n = \phi - n_g$ accounts for all other effects.

$$n_g(w(t)) \triangleq \sum_{i=1}^{N_T} n_{g_i}(w(t)) \triangleq \sum_{i=1}^{N_T} A_i^{\pm}(w(t))\delta(w(t) - T_i).$$

$$A_i^{\pm}(w(t)) \triangleq \begin{cases} 0 & w(t-\tau) = w(t) \\ A_i^{-} & w(t-\tau) > w(t) \\ A_i^{+} & w(t-\tau) < w(t) \end{cases}$$

Dynamic Glitch Nonlinearity n_g

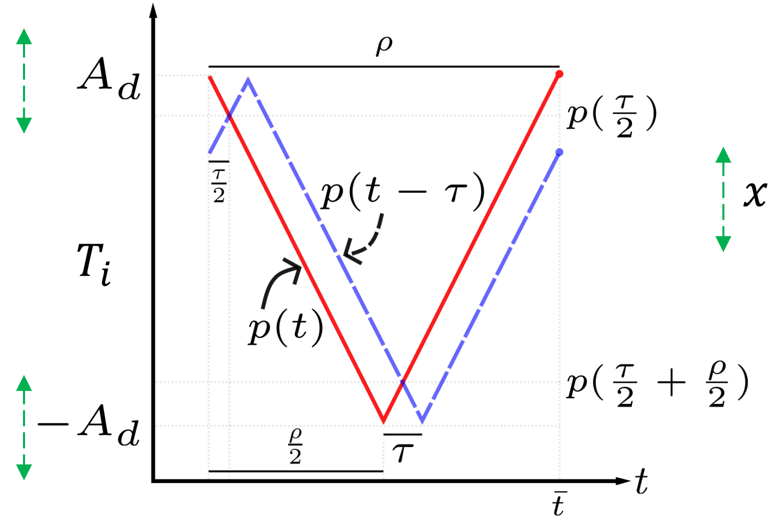




- δ is the Dirac delta function.
- T_i is a threshold value where a glitch is triggered and $T_i = T_j$ only when i = j. Thus, n_{g_i} describes the single glitch related to transition level T_i.
- N_T is the number of glitch transitions T_i.
- $A_i^{\pm}$ is the (input dependent) glitch area. Where $A_i^{+}(A_i^{-})$, indicate net areas of impulse responses for glitches due to references of rising (falling) trend.
- $\tau > 0$ is a finite time-delay. For a DAC, τ is limited by the duration between two consecutive input writes.

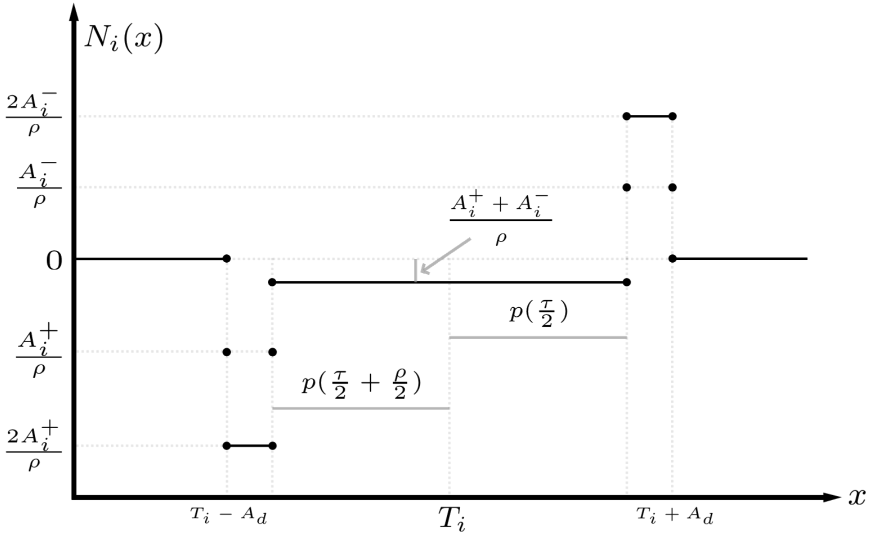
DITHERING A SINGLE GLITCH

$$N_i(x) \triangleq \frac{1}{\rho}\int_{[0,\rho)} n_{g_i}\left(x + p(t)\right)\mathrm{d}t = \frac{1}{\rho}\int_{[0,\rho)} A_i^{\pm}(x+p(t))\delta(x+p(t)-T_i)\mathrm{d}t \quad x \in \mathbb{R}.$$



- For a triangular wave dither $p(t)$ of amplitude A_d and period ρ:
- Clearly $N_i(x) \neq 0$ iff $x + p(t) - T_i = 0$. Implying that $x \in [T_i - A_d, T_i + A_d,] \triangleq I_x$.
- To calculate the value of $N_i(x)$ (for $x \in I_x$) first note that $A_i^{\pm}(x+p(t)) = A_i^{\pm}(p(t))$. $A_i^{\pm}(w(t)) \triangleq \begin{cases} 0 & w(t-\tau) = w(t) \\ A_i^{-} & w(t-\tau) > w(t) \\ A_i^{+} & w(t-\tau) < w(t) \end{cases}$
- Hence, we need to find the **sign** of $p(t) - p(t-\tau)$ at the **one** or **two** time instances where $p(t) = T_i - x$.

$$N_i(x) = \frac{1}{\rho}\begin{cases} 0 & \text{for} \quad x < T_i - A_d \\ A_i^{+} & \text{for} \quad x = T_i - A_d \\ A_i^{+} + A_i^{+} & \text{for} \quad T_i - A_d < x < T_i - p\left(\frac{\tau}{2}\right) \\ 0 + A_i^{+} & \text{for} \quad x = T_i - p\left(\frac{\tau}{2}\right) \\ A_i^{-} + A_i^{+} & \text{for} \quad T_i - p\left(\frac{\tau}{2}\right) < x < T_i - p\left(\frac{\tau}{2}+\frac{\rho}{2}\right) \\ A_i^{-} + 0 & \text{for} \quad x = T_i - p\left(\frac{\tau}{2}+\frac{\rho}{2}\right) \\ A_i^{-} + A_i^{-} & \text{for} \quad T_i + A_d > x > T_i - p\left(\frac{\tau}{2}+\frac{\rho}{2}\right) \\ A_i^{-} & \text{for} \quad x = T_i + A_d \\ 0 & \text{for} \quad x > T_i + A_d \end{cases}$$



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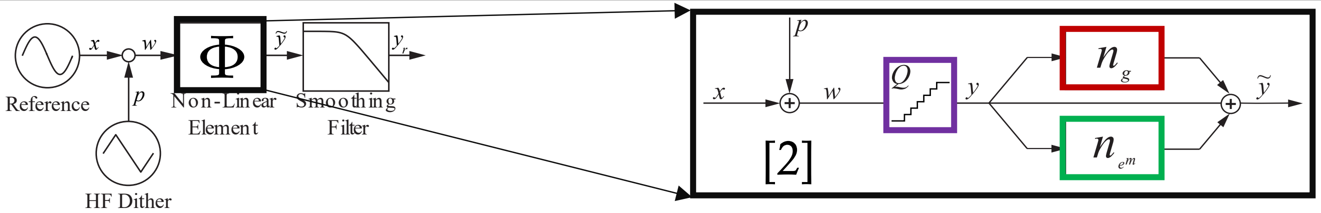
A VALIDATION MODEL

Comparison to a measured-response-based validation model



Non-linear DAC model accounting for glitched and INL non-linearities for a 16-bit Texas Instruments DAC8544.

Uniform Quantization

$$Q(w) = \delta T(w) = \delta \left\lfloor \frac{w}{\delta} + \frac{1}{2} \right\rfloor$$

$\delta = \frac{\Delta}{2^B - 1}$: LSB

Δ =5 to -5 V: FSR

$$n_{em} = \delta \mathrm{INL}(w)$$

$$\mathrm{INL}(w) \triangleq \frac{\tilde{y}(w) - \delta T(w)}{\delta}$$

Element mismatch Nonlinearity [6]

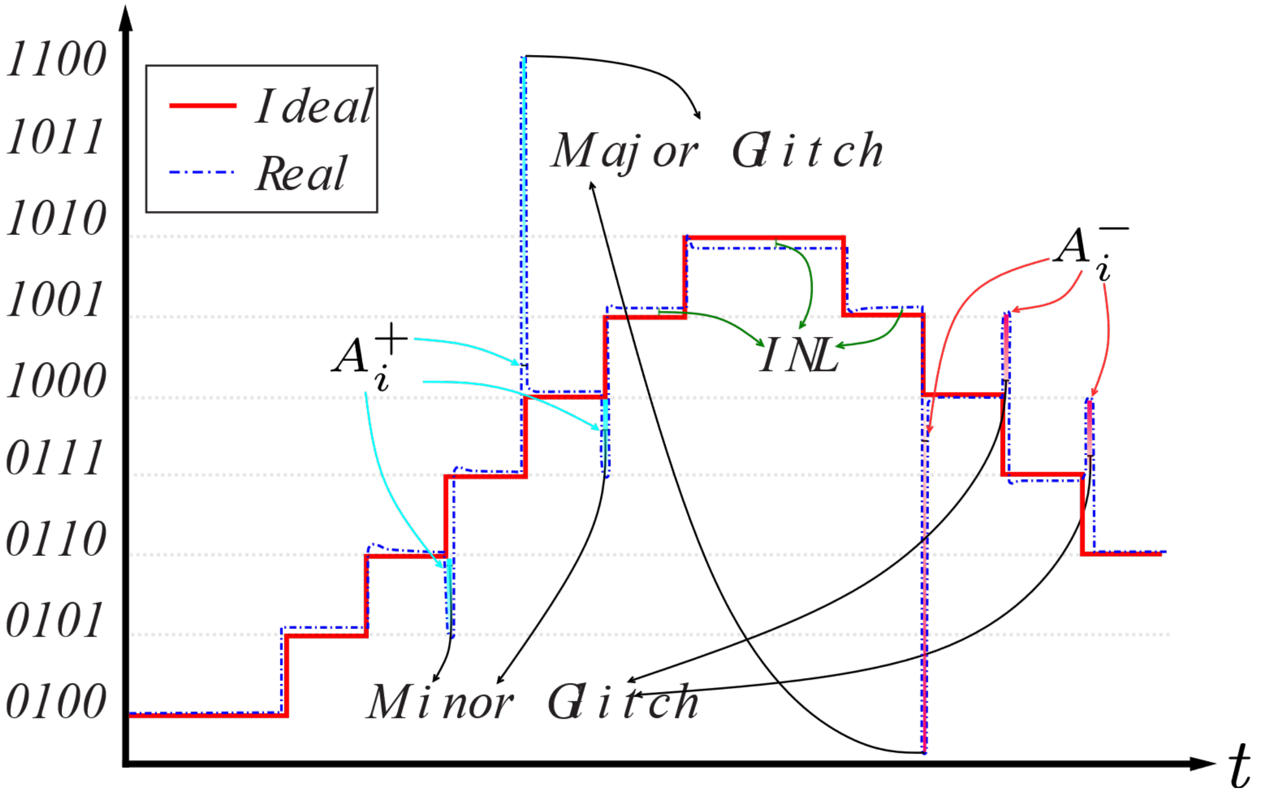
Novel A-/Symmetric Glitch Model [1]

Asymmetric $(T_j; j \in 1 \ldots N_{T_{\tilde{s}}} = 16)$: $A_i^+ = -60.6nVs, A_i^- = 51.9nVs$

Equidistant: $\Delta T_{\tilde{s}}$= 4096 LSB

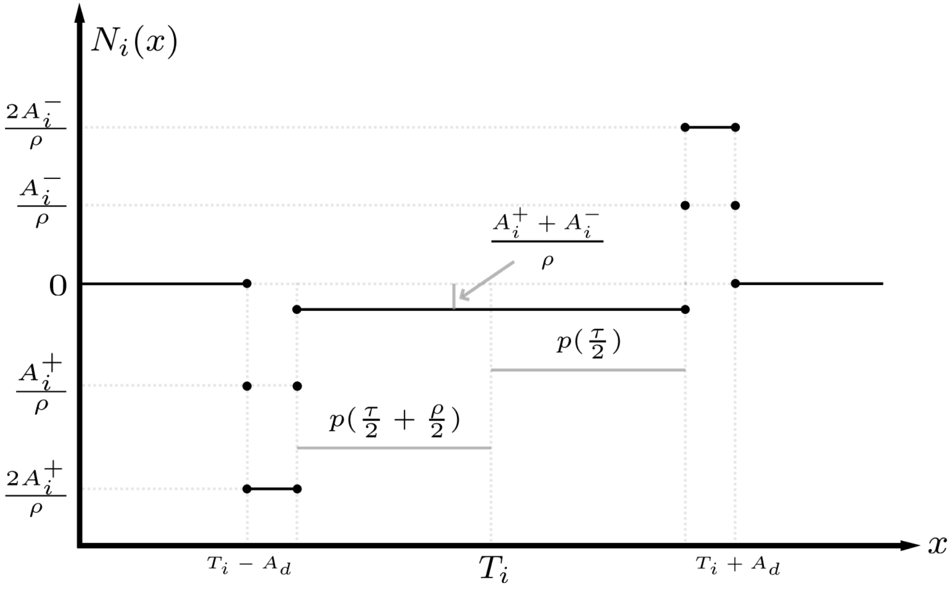
Symmetric $(T_i; i \in 1 \ldots N_{T_s} = 2^{16} - N_{T_{\tilde{s}}})$: $A_i^+ = -A_i^- = \pm 2.40nVs$

Equidistant:ΔT_s= 1 LSB

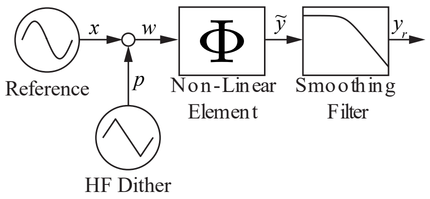


GLITCH MODEL VERIFICATION

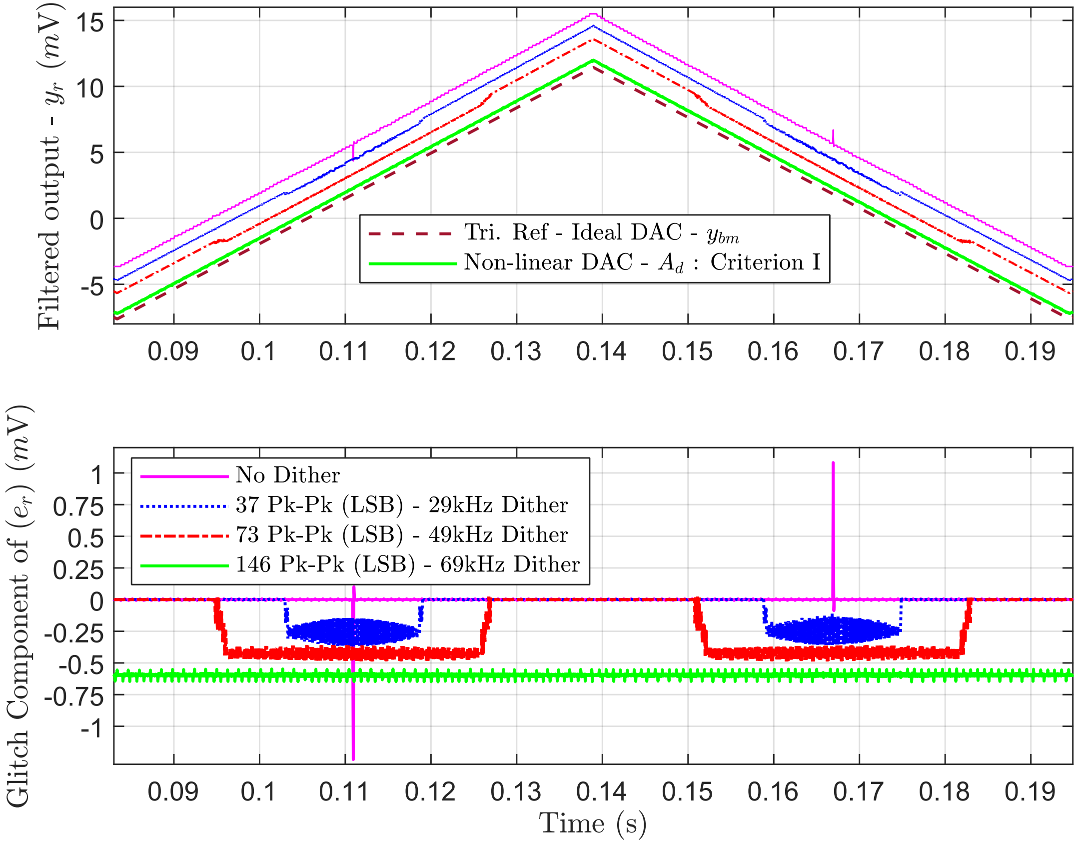
Note that the effect of progressively increasing A_d is to widen the sector where the averaged response N_i is a constant equal to $N_i(x) = (A_i^+ + A_i^-)/\rho = (-60.6 + 51.9)/\rho = -8.6363/\rho$ nV; i.e., -0.25, -0.423, -0.596 mV for $1/\rho$ = 29, 49, 69 kHz respectively.



Note that there seems to be a **sufficiently large** value for A_d where the averaged response $N_i(x(t))$ is rendered **constant** for $t \geq 0$.



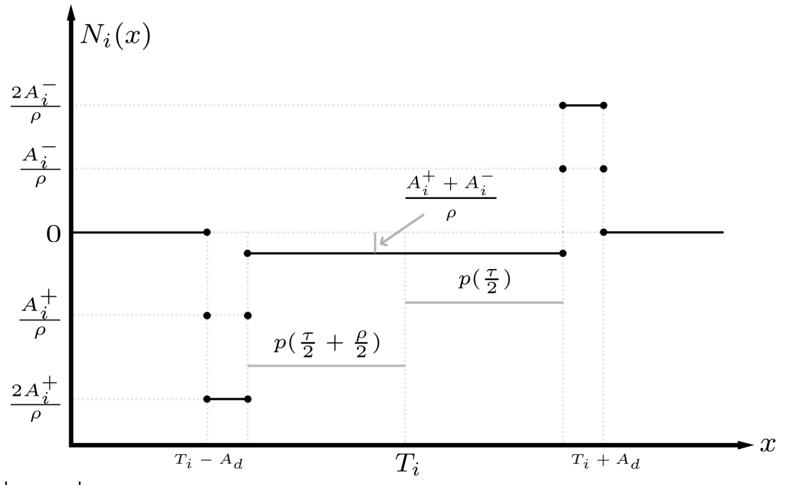
Let $y_{bm} \triangleq y_r$ when $\tilde{y} = x$. (Ideal DAC)
Then $e_r \triangleq y_r - y_{bm}$. (Residual error)



A CRITERION TO MITIGATE A SINGLE GLITCH

[5]

It is evident that $n_{g_i}(x(t))$ only depends on $x(t)$ when T_i is in the range of $x(t)$, that is, when $T_i \in [X_{min}, X_{max}]$



$$T_i - p\left(\frac{\tau}{2}\right) < x < T_i - p\left(\frac{\tau}{2} + \frac{\rho}{2}\right)$$

$$-A_d\left(1 - \frac{2\tau}{\rho}\right) < T_i - x < A_d\left(1 - \frac{2\tau}{\rho}\right)$$

$\Downarrow$

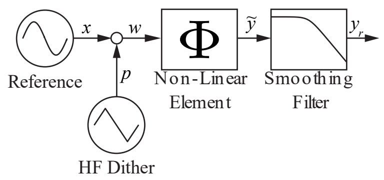
$$N_i(x) = (A_i^- + A_i^+)/\rho \quad \text{for all} \quad A_d > \rho/(\rho - 2\tau)|T_i - x|$$

$\Downarrow$

A_d can be chosen such that $N_i(x)$ is **constant** for all $x \in [X_{min}, X_{max}]$ and in turn $N_i(x(t))$ is **constant** for $t \geq 0$.

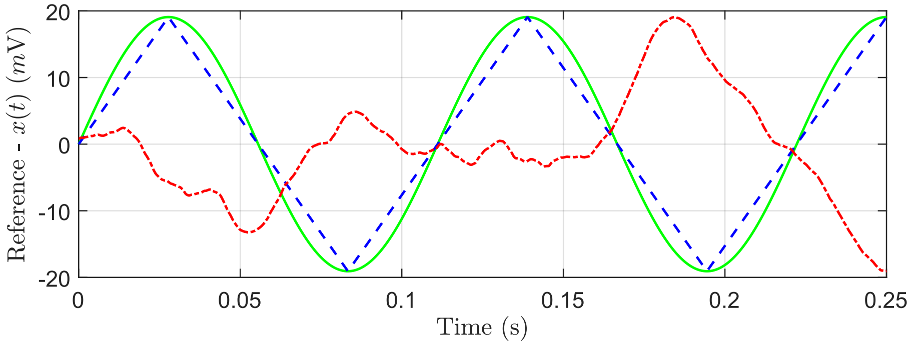
Criterion I: $$A_d > \left(\frac{\rho}{\rho - 2\tau}\right) \max\{|T_i - X_{\min}|, |T_i - X_{\max}|\}$$

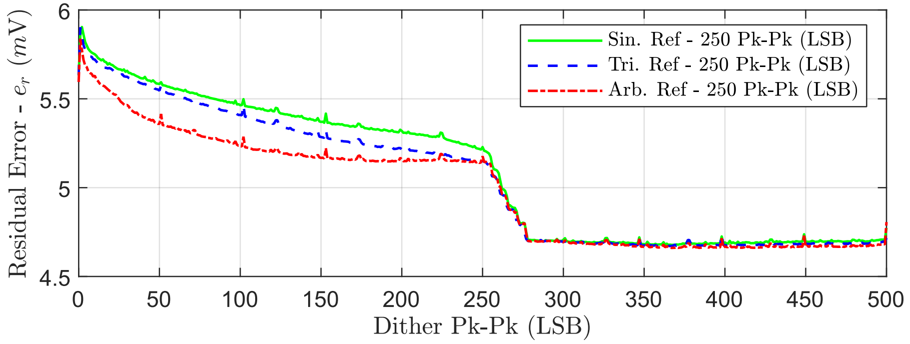
$$2A_d > 2 \cdot 1.109 \cdot max(|0 - -125|, |0 - 125|) = 277LSB$$



$y_{bm} \triangleq y_r$ when $\tilde{y} = x$. (Ideal DAC)
$e_r \triangleq y_r - y_{bm}$. (Residual error)

Various realisations of $x(t)$ within the range $x \in [-125,125]$ LSB: 9~Hz sinusoidal, 9~Hz triangular, and an arbitrary random walk. $T_i = 0$ LSB, $\frac{1}{\rho} = 49$ kHz, $\frac{1}{\tau} = 1$ MHz.





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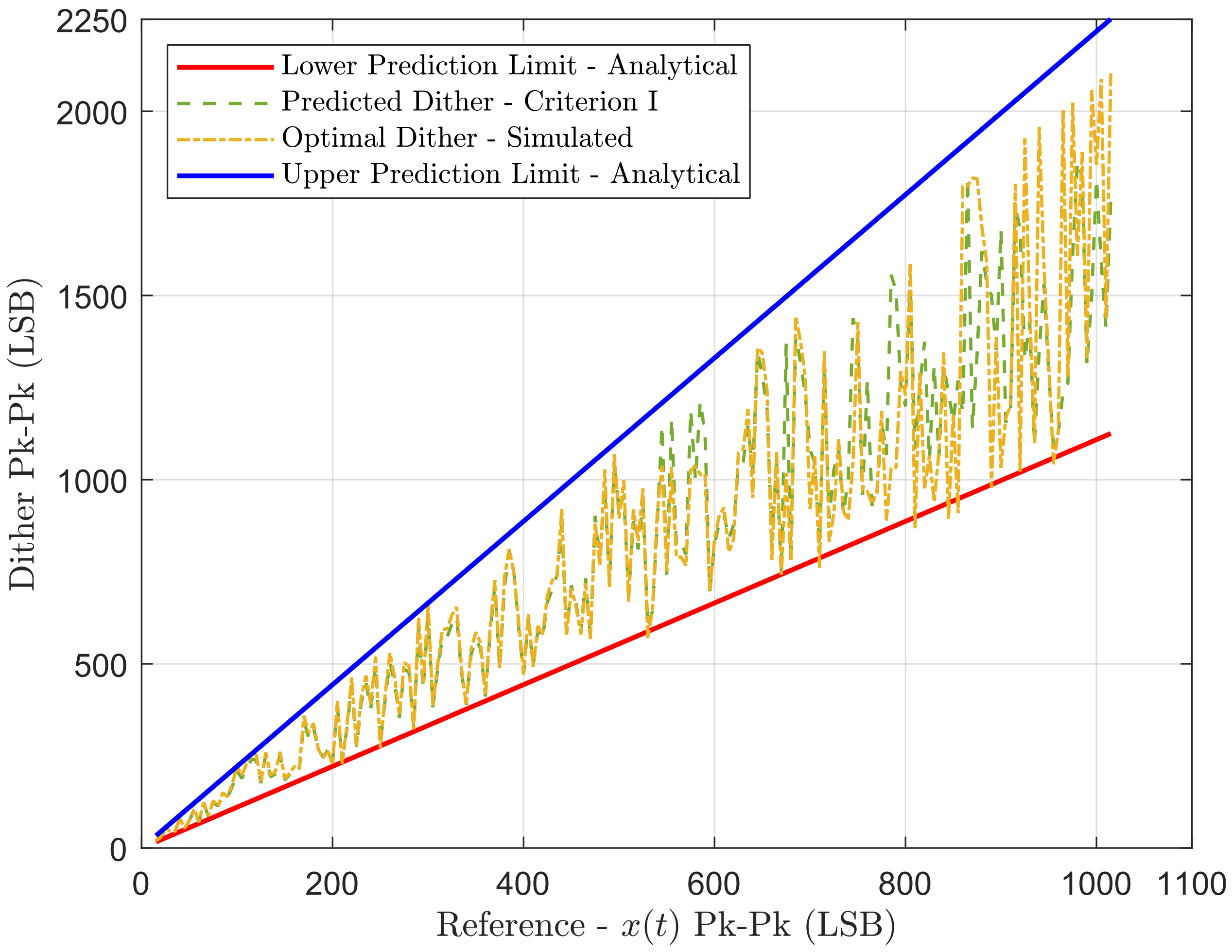
VALIDATING CRITERION I OVER A VARYING RANGE INPUT

- Validating Criterion I at $n_{g_i}(x(t))$ for an extended range of random walks at various ranges where $T_i = 0\ LSB$ is always in the range of $x(t)$. By comparing A_d as predicted in Criterion I due to varying X_{min}, X_{max}, against $A_{optimal}$ found when seeking the dither amplitude to render $N_i(x)$ constant over the range of x from simulation.
- For all T_i in the range of $x(t)$, the smallest dither amplitude to render e_r constant is bound between $\frac{\Delta X}{2}$ (lower prediction limit) and ΔX (upper prediction limit)



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A CRITERION TO MITIGATE MULTIPLE EQUIDISTANT GLITCHES

- For $A_d \geq \frac{\Delta T}{2}$ there will be an overlap between $N_i(x)$, $N_{i-1}(x)$, meaning Criterion I renders $N_g(x)$ constant for all $x \in [X_{min}, X_{max}]$ when $x \in [X_{min}, X_{max}] \subseteq [T_i - \left(\frac{(\rho-2\tau)\Delta T}{2\rho}\right), T_i + \left(\frac{(\rho-2\tau)\Delta T}{2\rho}\right)]$. **(General case limit)**

- An alternative trivial solution would be to take the maximum A_d from Criterion I over all T_j:
  $\max_{j\in\{1,2,3,\ldots N_{\{T\}}\}} \{A_d(j)\}$ where $A_d(j) = \left(\frac{\rho}{\rho-2\tau}\right) \max\{|T_j - X_{min}|, |T_j - X_{max}|\}$. **(Practical limitations)**

- Note the **special case** for when $A_{j-1}^{\pm} = A_j^{\pm} = A_{j+1}^{\pm}$, we have $N_i(x) = N_i(x + (j-i)\Delta T)$ for $j > i$. One can consider merging the side flanks of consecutive single glitches.

- A_d can be chosen such that it stitches neighboring glitches to maximize the range of x where $N_g(x)$ is **constant**.

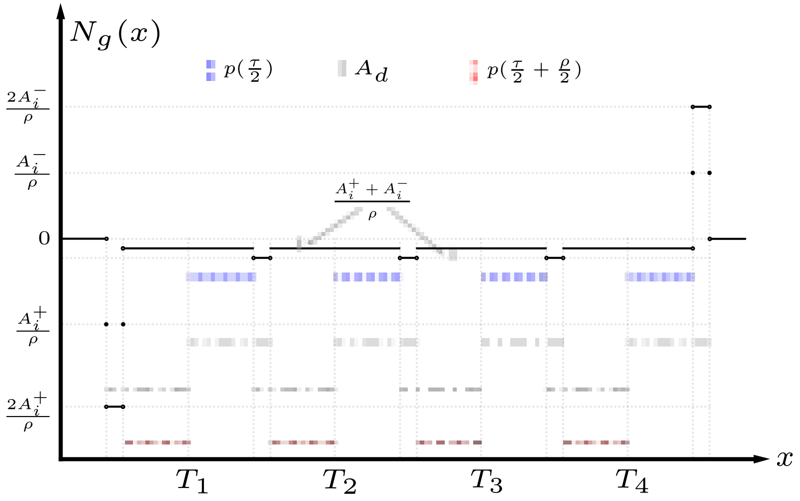
$$p\left(\frac{\tau}{2}\right) + A_d = T_i - T_{i-1} = T_{i+1} - T_i = A_d - p\left(\frac{\tau}{2} + \frac{\rho}{2}\right)$$

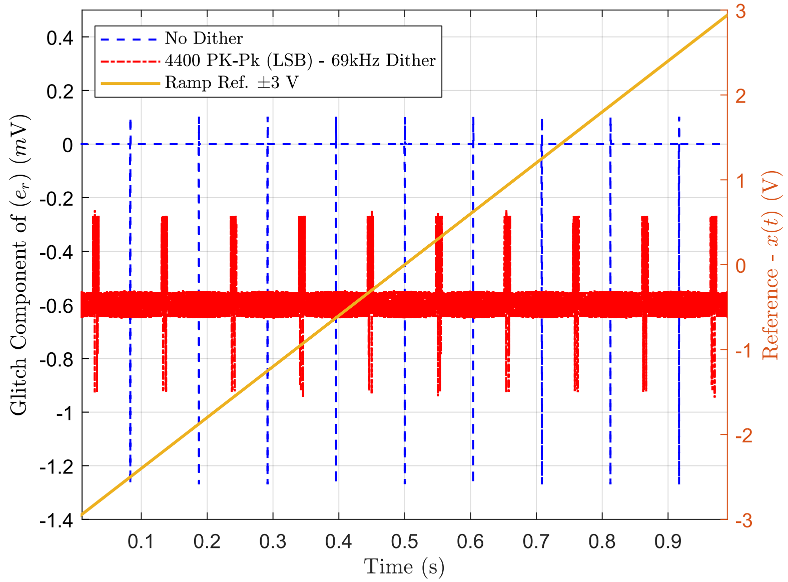
$$\Rightarrow \Delta T = 2A_d\left(1 - \frac{\tau}{\rho}\right)$$

Criterion II:

$$A_d = \left(\frac{\rho}{\rho-\tau}\right)\frac{\Delta T}{2}$$

$$2A_d = 2\left(\frac{\rho}{\rho-\tau}\right)\frac{\Delta T_{\tilde{S}}}{2} = 1.07411 \cdot \frac{4096}{2} = 4400LSB$$





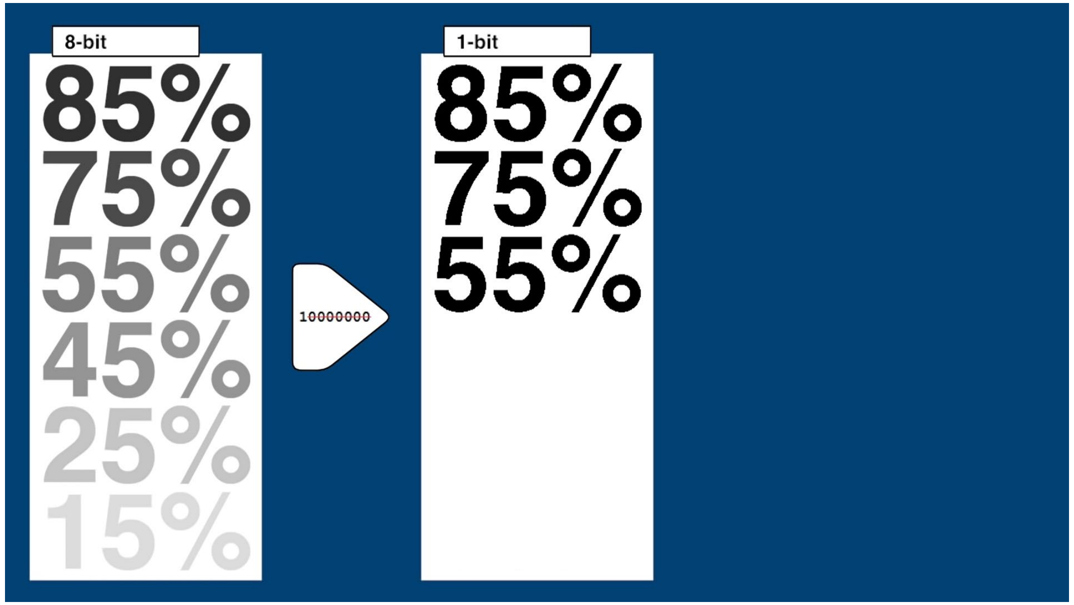
REFERENCES

- [1] Eielsen, A. A., Leth, J., Fleming, A. J., Wills, A. G., & Ninness, B. (2020). Large-amplitude Dithering Mitigates Glitches in Digital-to-analogue Converters. *IEEE Transactions on Signal Processing*, 68, 1950-1963.
- [2] Faza, A., Eielsen, A. A., Leth, J. Mitigating Non-linear DAC Glitches Using Dither in Closed-loop Nano-positioning Applications. *In 2023 American Control Conference (ACC)* . (pp. 685-691). IEEE.
- [3] Wannamaker, R. A., Lipshitz, S. P., Vanderkooy, J., & Wright, J. N. (2000). A theory of nonsubtractive dither. *IEEE Transactions on Signal Processing, 48*(2), 499-516.
- [4] Zames, G., & Shneydor, N. (1976). Dither in nonlinear systems. *IEEE Transactions on Automatic Control*, *21*(5), 660-667.
- [5] Iannelli, L., Johansson, K. H., Jönsson, U. T., & Vasca, F. (2006). Averaging of nonsmooth systems using dither. *Automatica*, 42(4), 669-676.
- [6] Eielsen, A. A., & Fleming, A. J. (2017). Improving digital-to-analog converter linearity by large high-frequency dithering. *IEEE Transactions on Circuits and Systems I: Regular Papers, 64*(6), 1409-1420.

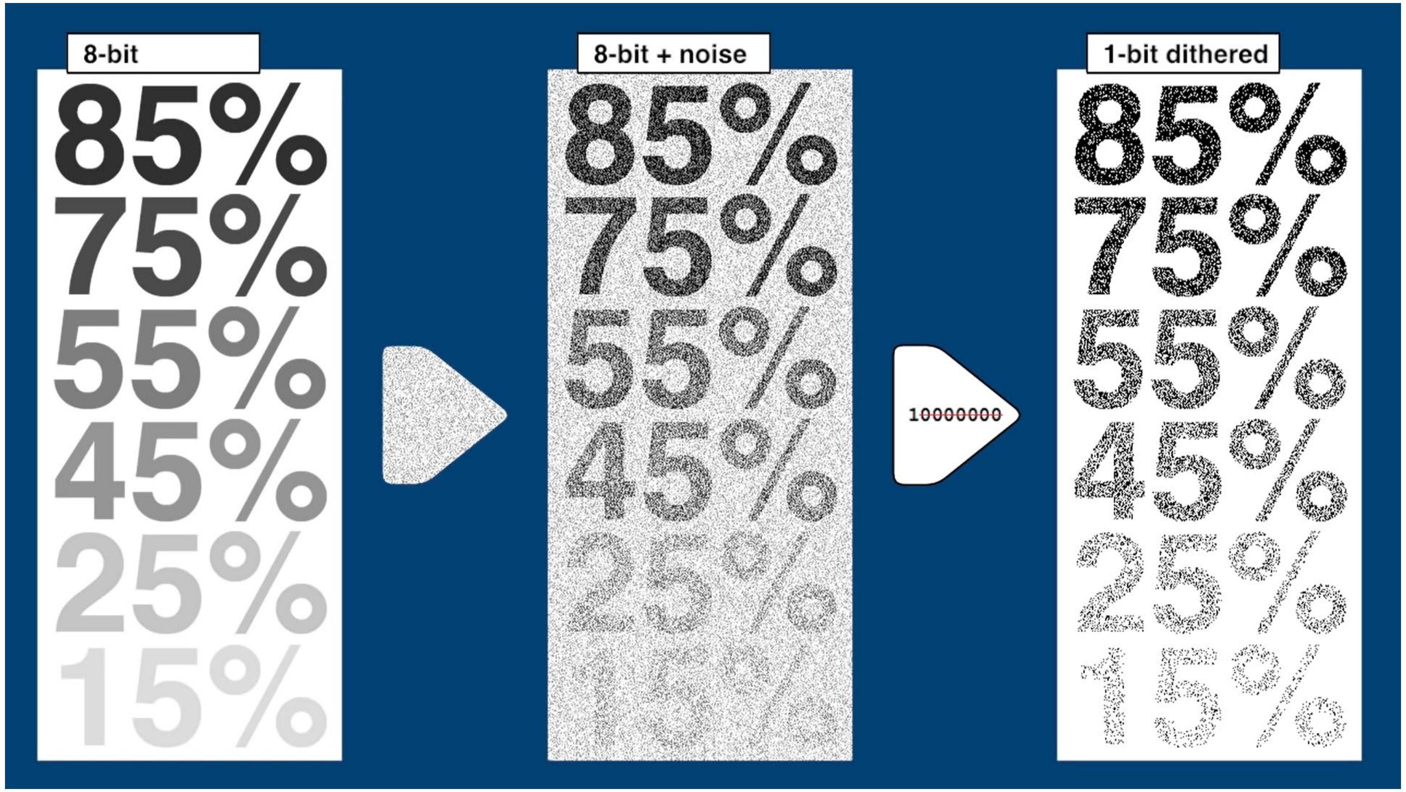
THANK YOU

DITHERING EXAMPLE

Ideal Quantization

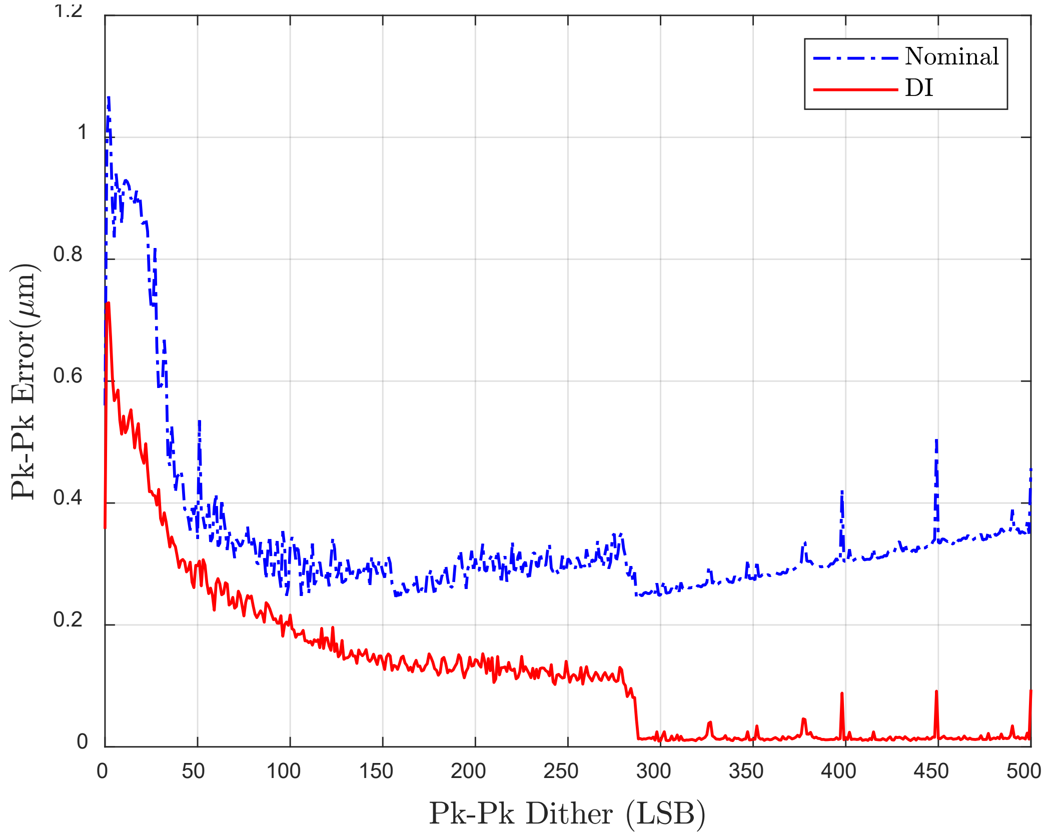


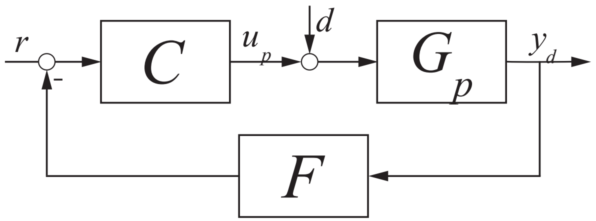
Dithered system

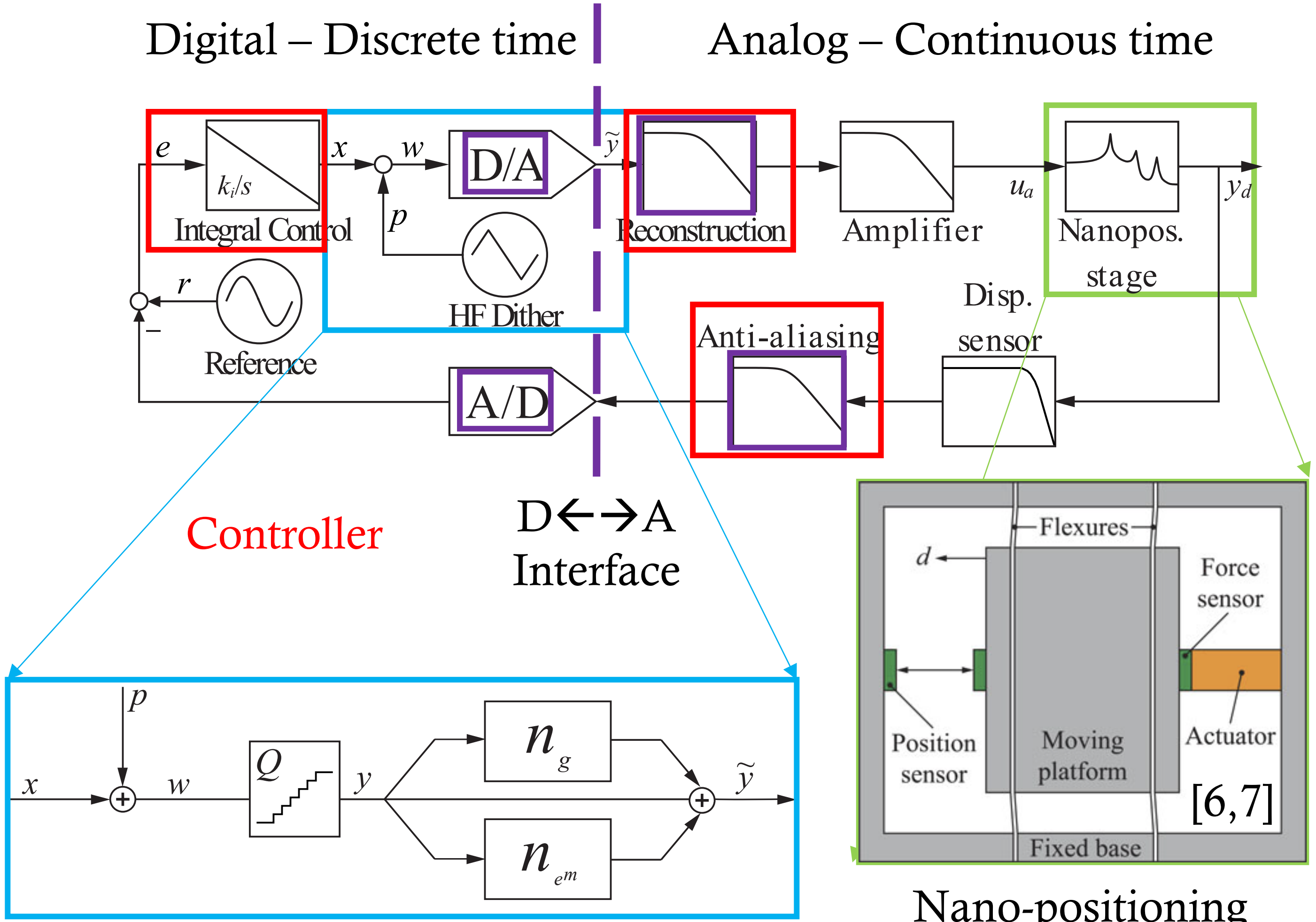


CLOSED-LOOP NANO-POSITIONING SYSTEM

[5]







DAC model accounting for glitched and INL non-linearities

Nano-positioning mechatronic system