Linearisation of Digital-to-Analog Converters using Model Predictive Control

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Introduction

- **Digital to Analog Converters**
  - Key to bridging the analog and digital domain,
  - Converts digital code to analog voltage or current,
  - Analog output is proportional to the digital input.
- **Applications**
  - **Digital Audio**: CD Players, telephones
  - **Function Generators:** Digital oscilloscopes, signal generators
  - Motor controllers
  - Interferometry

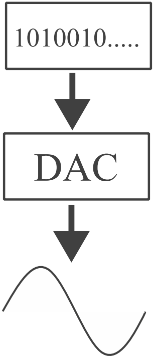

Figure: Digital-to-analog conversion

Introduction

- DAC: Quantisation (Re-quantisation) and Reconstruction filter
- Quantisation → quantisation error and static error(INL)
- adds noise and distortions to the analog output signal
- method to mitigate the effects of these errors

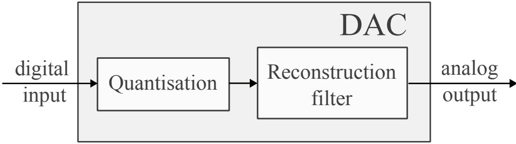

Figure: DAC

Quantisation

- **Quantisation (Re-quantisation)**: map analog infinite (finely quantised) input values to a smaller set of discrete finite values,

- Quantisation:

$$y = \mathbf{Q}(w) = \delta\ell := \delta\left\lfloor \frac{w}{\delta} + \frac{1}{2} \right\rfloor, \qquad (1)$$

  $\lfloor . \rfloor$ is a floor operator.

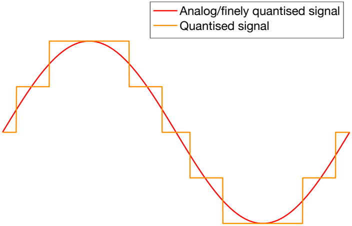

Figure: Analoge sinusoidal and quantised signal.

Quantisation Error

- **Quantisation**
  - ➜ Analog signal is represented by the digital number of finite number of bits,
  - ➜ Introduce loss of information.
- **Quantisation Error:** difference between input and quantised output

$$q = y - w. \tag{2}$$

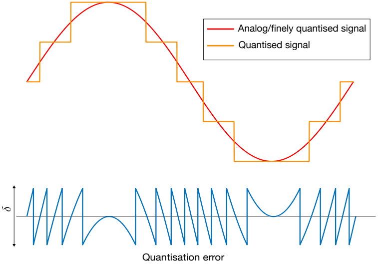

Figure: Quantisation and quantisation error

- **Resolution**: amount of variance in output value for every change of the value in the digital input,
- $\mathcal{B}$ bit → $2^{\mathcal{B}}$ levels (4 bit → 16 levels),
- Introduces noise and distortion in the output signal.

Static Error: Integral Non-linearity (INL)

- **Static errors**: deviation of actual levels from the ideal ones in the DAC, due to element mismatch.
- **Integral non-linearity (INL):**
  - $y-$ ideal output,
  - $\mathrm{INL}(\ell)-$ deviation at the level $\ell \in \mathbb{N}$,
  - $\tilde{y}-$Actual output,

$$\tilde{y}(\ell) = y(\ell) + \delta INL(\ell). \quad (3)$$

- cause unwanted noise and disturbance,
- limits the accuracy of the analog output signal.

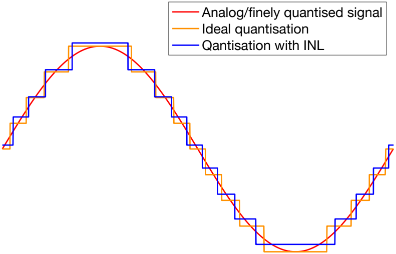

Figure: Quantisation: Ideal and non-ideal (actual)

Moving Horizon Optimal Quantiser [Goodwin et. al. 2003][1]

- Quantisation problem is cast into the general moving horizon optimisation problem.
- Obtained by incorporating the reconstruction filter $H(z)$ into the model

$$H(z) = 1 + C(zI - A)^{-1}B$$

- State-space realization of the system:

$$\begin{aligned} x(t+1) &= Ax(t) + B(y(t) - w(t)) \\ e(t) &= Cx(t) + y(t) - w(t) \end{aligned} \tag{4}$$

- Error e represent the difference between the filtered reference signal and filtered quantised signal.

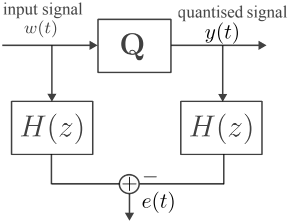

Figure: Incorporation of the filter

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[1] Goodwin, G.C., Quevedo, D.E., and McGrath, D. (2003). Moving-horizon optimal quantizer for audio signals.Journal of the Audio Engineering Society, 51(3), 138– 149.

Moving Horizon Optimal Quantiser

- Problem Formulation:

$$y^*(t) = \arg\min_{y(t)} V_N = \sum_{t=k}^{k+N-1} e^2(t) \tag{5a}$$

subject to

$$x(t+1) = Ax(t) + B(y(t) - w(t)) \tag{5b}$$

$$e(t) = Cx(t) + y(t) - w(t) \tag{5c}$$

$$y(t) \in \mathbb{U}^N. \tag{5d}$$

where,

$\mathbb{U} = \{y_1, y_2, \ldots, y_{n_{\mathbb{U}}}\}$, $n_{\mathbb{U}} = 2^{\mathcal{B}-1}$ is a discrete set representing quatisation levels,

N - finite prediction horizon,

$\mathbb{U}^N := \mathbb{U} \times \ldots \times \mathbb{U}$ and contains $n_{\mathbb{U}}^N$ elements.

Moving Horizon Optimal Quantiser

- Closed-form solution is provided in [Goodwin et. al 2003][1] but
  - assumes ideal uniform quantisation model,
  - transforms to vector quantisation problem,
  - the dimension of the solution space depends on the of quantiser resolution and the prediction horizon length.
- In practice DACs are
  - non-uniform thus the method provides no linearising effect in practice and,
  - large solution space makes the approach impractical for the precision DACs with higher number of quantisation levels.
- Modified MHOQ implementation method:
  - non-ideal DACs, include INL into the model,
  - solve MHOQ numerically as mixed-integer scalar quantisation problem.

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[1] Goodwin, G.C., Quevedo, D.E., and McGrath, D. (2003). Moving-horizon optimal quantizer for audio signals.Journal of the Audio Engineering Society, 51(3), 138– 149.

Moving Horizon Optimal Quantiser with INL

- Block diagram of implementation is shown in the following figure

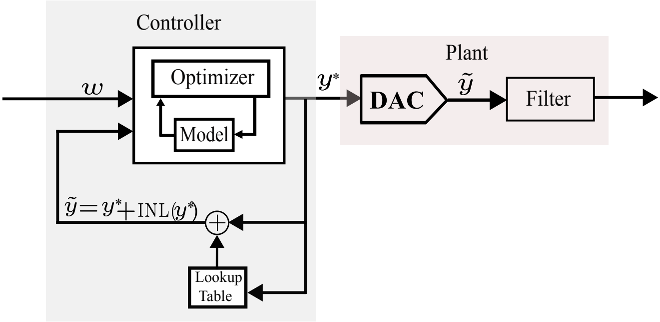

Figure: Synthesis and implementation of moving horizon optimal quantisers in DAC with INL.

Simulation

- Signal to noise and distortion ratio (SINAD) $= 20\log_{10}\left(\frac{\sigma_s}{\sigma_t}\right)$

  where, $\sigma_s =$ s.d. of the input signal,
  $\sigma_t =$s.d. of other unwanted components in the output signal

- DAC Model: model used in designing the control scheme
- DAC Actual: model used as actual converter response

Quantisation Method	DAC Model	DAC Actual	Abbrev.
Direct	Without INL	Without INL	DQ-1
Direct (Baseline)	Without INL	With INL	DQ-2
MHOQ (Ideal)	Without INL	Without INL	MHOQ-1
MHOQ (Goodwin et. al.)	Without INL	With INL	MHOQ-2
MHOQ (This paper)	With INL	With INL	MHOQ-INL

Table: Abbreviations for different combinations of method, DAC model used in control scheme, and actual DAC response.

Simulation and Experiment: Setup

- Reference signal: $w(t) = A\sin(2\pi ft)$
- Carrier signal frequency $f = 1$ kHz
- Sampling frequency $f_s = 1$ MHz
- Second-order Low-pass butterworth filter $H(z)$ with cutoff frequency $f_c = 10$ kHz.
- 8- bit DAC
- INL- measured form the actual DAC

- Experimental Setup

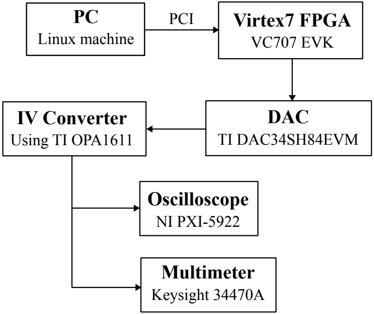

Figure: Experimental setup

Simulation and Experiment: Results

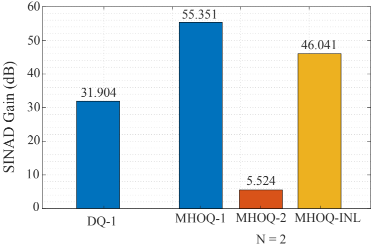

Figure: Simulation SINAD gains for various methods, relative to the baseline ($\mathrm{SINAD}_{\text{DQ-2}} = 41.236$).

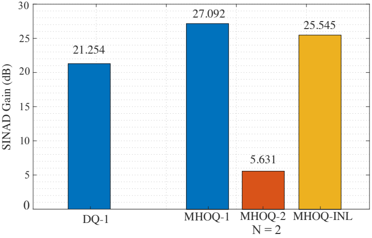

Figure: Experimental SINAD gains for various methods relative to the baseline ($\mathrm{SINAD}_{\text{DQ-2}} = 41.470$).

Conclusion

- A modified MHOQ based approach proposed to minimise error in DACs.
- INL is integrated into the model as a lookup table.
- The proposed method was validated through simulations and experiments on a common off-the-shelf DAC.
- Incorporating INL into the model using MHOQ improves DAC performance.